Contribution on the Control of Mixed Constrained Discrete Event Systems for a Flexible Workshop Application

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***Abstract*—This paper aims to extend the existing developed approaches for the control synthesis of state feedback control for the class of Timed Discrete Event Systems particularly its subclass represented by Timed Event Graphs. Accordingly, we introduce a new solving approach based on the Petri nets under the existence of mixed constraints. Capacity and time are both critical criteria that could sometimes evolve together and need to be respected at the same time for proper systems conduction, especially for industrial sensitive applications. In this sense, we look to compute suitable control laws in order to meet these critical specifications. Wherefore, we aim to satisfy sufficient conditions to ensure the respect of these restrictions. Based on the use of linear dioid algebra for the analytical modeling and on Petri nets for the graphical modeling, the general dynamical behavior of the system is described by linear Min-plus equations while the mixed constraints are represented by linear inequations.**

***Index Terms*—Discrete Event Systems (DES), Petri nets, Timed Event Graph (TEG), dioid algebra min-plus, control synthesis, strict mixed constraints**

I. Introduction

Min-plus algebra is a framework of linear algebra over Min-plus semiring, which is defined by the set $\overline{\mathbb{R}}_{\min} = (\overline{\mathbb{R}} \cup \{-\infty\}, \oplus, \otimes)$, and characterized with both operations; multiplication $a \otimes b = a + b$ and addition $a \oplus b = \min(a,b)$. This idempotent semiring is characterized by neural element $\varepsilon = +\infty$ and unity $e = 0$. The Arithmetic operation in the sequel of Min-plus algebra is also extended to the case of matrix, in our case of concern, all matrix multiplications are in the Min-plus sense.

Petri Nets had proved their efficiency in representing a wide range of the Discrete Event Systems due to their simplicity for a various uses notability for system's analysis and control design. In the sequel of this paper we specifically use P-Timed Petri Nets where time delays are associated to places. In order to model linear systems we used a class of Petri Net such that all of its places have only one input transition and one output transition. This class is TEG. The linear representation is deduced worth to a set of counter functions according to the evolution of the system's status. For this purpose recurring linear equations are fixed to represent the behavior of the Timed Event Graph, and the inequations are used to describe the set of the mixed constraints on places. Strict specifications related to capacity and time are the crucial criterions that are needed to be respected for a proper system's conduction. This frequently encountered problem faced through many of the real industrial processes sensitive to these specifications catches our interest that fore we look to solve this issue by providing control laws involving delay and satisfying specific conditions.

This current contribution adapts existing approaches in order to solve the control problem issue under the existence of mixed constraints as well as introduces a generalization covering wider encountered applications within more complicated systems. We refer to our previous works in the same issue like in [1] where a control design was established to solve time constraints availability in paths. By relaxing some previous hypothesizes in the same topic, the presented solution proves its efficiency in the problem of possible loops without tokens and contributes on keeping the properties of the Timed Event graph.

In Section II we evoke the theoretical recalls required in this work notably Min-plus algebra formalism and with a focus on the TEG class. The following section is focused on the problem formulation by introducing the concept and the properties of the mixed constraints over Min-plus. In Section IV, we evoke the control synthesis resolution. Section V is reserved for the application case and finally the conclusion with some future perspectives.

II. Thoretical Recalls

An abundant literature was dedicated to introduce the Petri Nets framework, like in [2] and [3]. Since a Petri net is known as a biparti-graph, it is composed of places and transitions linked together with arcs. Each marked place contains a finite number of tokens that define the dynamic of the systems. Considering tokens dynamic

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inside the graph it is possible to model the state at a specific point of the systems evolution.

$P = p_1, p_2, ..., p_n$ stands for the set of n places.

$T = t_1, t_2, ..., t_m$ stands for the set of m transitions.

$\tau \in N^p$ defines the temporizations associated to places, they are the minimal durations for a token to move on to the next place.

In our case, we are interested particularly on ordinary P- Timed Petri nets where all the arcs linking places to transitions are 1 weighted.

We start with an over view of the matrix multiplication and the equation resolution over the dioid algebra Min-plus. We give the notations:

Let $A \in \mathbb{Z}_{\min}^{m\times p}, B \in \mathbb{Z}_{\min}^{p\times n}$, and $C \in \mathbb{Z}_{\min}^{m\times n}$.

The resulting matrix's multiplication like:

$$C_{ij} = (A \otimes B)_{ij} = \bigoplus_{k=1}^{p} A_{ik} \otimes B_{kj}$$

Remark 1 the matrix calculation considers specific properties such that:

- a, is a non-null integer.
- e, stands for the neutral element.
- ε, is the absorbing element, Like it is equals to $\varepsilon = +\infty$.

Definition 1 Let $H^* \in \overline{\mathbb{R}}_{\min}^{n\times n}$ is a square matrix called the Kleene star matrix over the smearing Min-plus. It is calculated with the formula: $H^* = \underset{i\in\mathbb{N}}{\oplus} H^i$ such that the matrix H^0 is called the unit matrix and it is characterized by diagonal entries equal to e and ε otherwhere.

A counter function associated to a transition t_j is an increasing application, denoted as $x_j(t)$ like $\mathbb{Z} \to \mathbb{Z} \cup \{\pm\infty\}, t \to x_j(t)$ $x_j(t) \in \mathbb{Z} \cup \{\pm\infty\}$ corresponds to the number of firing of the transition t_j until the date t it represents the cumulated number of firing of the transitions. $u(t) \in \overline{\mathbb{R}}_{\min}^{m}$ stands for counter functions related to source transitions, and $\theta(t) \in \overline{\mathbb{R}}_{\min}^{n}$ stands for those of any internal transition of the TEG. Among the first research in this filed, dealing with the dynamical behavior of Timed Event Graphs, we cite [4], where authors expose the implicit equation (1) that could be replaced by its equivalent explicit equation (2).

$$\theta(t) = \bigoplus_{\tau=0}^{\tau^{\max}} (\mathrm{A}_{\tau}.\theta(t-\tau) \oplus B_{\tau}.u(t-\tau)) \qquad (1)$$

$$\theta(t) = \bigoplus_{\tau>0} (A_0^*.A_{\tau}.\theta(t-\tau) \oplus A_0^*.B_{\tau}.u(t-\tau)) \qquad (2)$$

We give:

$\mathrm{A}_{\tau} \in \overline{\mathbb{R}}_{\min}^{n\times n}$ is a square matrix, in which its entries are defined by $\mathrm{A}_{\tau,ij}$ representing the initial marking m_{ij} of each place p_{ij}, if the place is not marked we note the corresponding entry like $m_{ij} = \varepsilon$.

$B_{\tau} \in \overline{\mathbb{R}}_{\min}^{n\times m}$ where the entries represent the marking of the places linked straight to the resource transition.

$\tau^{\max}$ represents the maximum delay arising in the Timed Event Graph. Such that: $\tau^{\max} = \max_{ij/p_{ij}\in P} \{\tau_{ij}\}$.

A_0^* is the Kleene star of A_0, as already same as defined.

We consider the earliest starting policy where each transition fires as soon as it is enabled.

A. State Space Model for TEG

Similar to the linear classical systems theory and in order to seek for the state space model equation:

$$x(t) = A.x(t-1) \oplus Bu(t) \qquad (3)$$

The concept consists in substituting each places on the TEG with a temporizations $\tau > 1$ and time units temporizations by a number of τ places and a number of $(\tau - 1)$ transitions. The resulting extended graph is characterized by a number of n' counter functions for these added transitions. As a result the state space vector is given as follows: $x(t) \in \mathbb{R}_{\min}^{N}$ like $N = n + n'$.

The dynamical behavior of the TEG could be expressed through the following equation:

$$x(t) = \widehat{A}_0^*.x(t) \oplus \widehat{A}_1^*.x(t-1) \oplus \widehat{B}u(t)$$

Like we have: $A = \widehat{A}_0^*.\widehat{A}_1^*$ and $B = \widehat{A}_0^*.\widehat{B}$.

It is equivalent to the sate space equation being given by equation (3).

We conclude from all the previous notations that the event graph is deterministic, depending basically on the input $u(t)$ and on some initial conditions. As this dependency could be explicit, we shall use the following equation:

$$x(t) = A^{\tau}.x(t-\tau) \oplus \left[\bigoplus_{k=0}^{\tau-1} A^k.B.u(t-k)\right] \qquad (4)$$

This equation holds true, for every $\tau \geq 1$.

III. Mixed Constraints Problem

A. Time Constriant

Temporal constraints are mostly common restrictions to a wide range of discreet event systems. Therefore, the respect of time in the control synthesis resolution is a serious research issue that has attracted the attention of researches' interest. For instance, in [5], authors look to determine the sizing and the control of the plant under an existent temporal constraint. Authors in [6], focus on automatic test systems in which they show the validation of systems that include timing constraints. An application of rail transport is deployed in order to illustrate their approach. In addition, similar to application in [7], authors take into account time constraints caused by the leading train through solving optimal control problem based on the use of two different approaches: the greedy and the simultaneous approach. In [8] and [9], their main interest was about real time constraints in order to find control processing times minimizing a cost function for each task subject of the constraints. Moreover, Also temporal constraints were the main concern for authors of [1] and [10], control laws with the use of both semiring of dioid algebra Min-plus and Max-plus were determined to ensure these specifications. It has shown in [11] the way timing issues are crucial especially for manufacturing systems, they proposed a formal method for the analysis and the control of P-Timed petri nets. Unlike some developed approaches listed under the framework of non-timed dynamic event systems as in [12], in our case, we are going besides to focus on the class of Timed Event systems with critical time, like the case of tasks delimited by a time bound. In the place p_{ij} , let $\tau_{ij}^{\min}$ is the minimal sojourn time of the token that is taking in advance by the linear model. Since $\tau_{ij}^{\min} = \tau_{ij}$, only the maximal time must not be exceeded that seems the additional temporal specification that we are aiming to satisfy.

Referring to [13] and [14], the expression of the temporal constraint is deduced from the following inequality:

$$x_i(t) \geq m_{ij0} x_j(t - \tau_{ij}^{\max}) \tag{5}$$

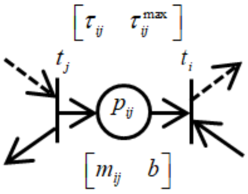

Figure 1. Mixed constraints on the place p_{ij}

B. Marking Constraints

Marking in terms of Petri Nets characteristics assigns to places a nonnegative integer which refers to tokens. Their dynamic follows some firing rules. For instance, they could be translated as the caring capacity of materials for stock areas, memory capacity in networked communication systems, products to be proceeded in an industrial production line. In literature, several attempts have revolved around the control synthesis resolution to satisfy marking constraints such as in [15] and [16], we find approaches based on the use of Petri Nets place invariants to synthesize control that consider constraints on the system's marking behavior, these constraints are linear inequalities based on elements of Petri Nets marking vectors. Even though the approach can be used to build Petri net controllers in a modular manner, it is not efficient for the case of uncontrollable transitions. Recently in [17], an approach has showed an interest in the places with critical marking. Authors explored the Generalized Mutual Exclusion Constraints (GMECs), through using resource-based observers in order to approximate the maximum markings of critical places. Another attempt in [18] to resolve the problem of marking restriction. On which the objective of the developed control design is to restrict the number of tokens at certain places with the use of both Min-plus algebra and TEGs. Accordingly, authors established some relaxing hypothesizes. Although their approach was applied to a manufacturing line and to an assembly system, the proposed formulation does not consider the issue of unobservable transitions on the TEG.

Assuming that the place illustrated in Fig. 1 is assigned to indicate the maximal marking to be tolerated. Let m_{ij0} be the initial marking of this place, $x_j(t)$ is the counter function assigned to the transition t_j up to t time and m_{ij} represents the available marking in the place p_{ij} at t time which is equivalent to $x_j(t) - x_i(t) + m_{ij0}$. According to [18], the marking is represented by the inequality $m_{ij}(t) \leq b$ is equivalent to:

$$x_j(t) - x_i(t) + m_{ij0} \leq b$$

Which could be transformed to the equivalent following Min- plus inequality of the constraint:

$$x_j(t) \leq (b - m_{ij0}).x_i(t) \tag{6}$$

IV. Control Formulation over Dioid Min-plus

In this section, we address the control formulation step in order to find suitable control laws in the case of mixed constraints on places. Thereupon, we introduce the Theorem 1, which gives an overview of the control synthesis in the case of the existence of single mixed constraints.

Taking equation (4) into consideration, if we substitute the parameter τ , it will be substituted by ϕ . Hence, we get the equation (7) as follows:

$$x_i(t) = \left[\bigoplus_{r=1}^{N} (A^{\phi})_{ir} x_r(t-\phi)\right] \oplus \left[\bigoplus_{k=0}^{\phi-1} (A^k.B)_i.u(t-k)\right] \tag{7}$$

Such that the parameter ϕ is $\phi \geq 1$ and it is equals to $\phi = \phi_x$ in the case of the time constraint and $\phi = \phi_y$ is substituted when it consists of capacity constraint.

Having $A \in \overline{\mathbb{R}}_{\min}^{n\times n}$ and $B \in \overline{\mathbb{R}}_{\min}^{n\times m}$. Such that n denotes the number of the internal transitions and m denotes the number of the input transitions. Accordingly $(A^{\phi})_{ir}$ designs the i^{th} components of the matrix A^{ϕ} .

Let α denotes the path delimited by the resource transition and the upstream transition of the place under mixed constraint. We define by τ_{α} and m_{α} successively; the sum of all time delays and the sum of the markings.

As $x_j(t)$ represents the counter function of the upstream transition of the constrained place, and $u_x(t)$ represents the counter function of the resource transition. Accordingly, we provide the following inequation:

$$x_j(t) \leq m_{\alpha}.u_x(t - \tau_{\alpha}) \tag{8}$$

This inequation represents the key to achieve the following results.

Remark 3 we admit that $u_x(t)$ stands for the control that satisfies time constraint case and $u_y(t)$ is the control in the case of marking constraint.

Theorem. 1.

A TEG whose evolution is given by the equation (3), is subject to both time and capacity constraints on the place p_{ij} respectively of the form (5) and (6). It admits a control law $u(t)$ of the form:

$$u(t) = \min(F_x x_r(t-1), F_y x_r(t-1)) = (u_x(t) \oplus u_y(t))$$
$$= F_x x_r(t-1) \oplus F_y x_r(t-1) \quad (9)$$

Wit $F_x = [\overset{N}{\underset{r=1}{\oplus}}(A^{\phi_x})_{ir} - m_{ij}.m_\alpha]$, where $\phi_x = \tau_\alpha + \tau_{ij}^{\max} + 1$ and $F_y = [\overset{N}{\underset{r=1}{\oplus}} A^{\phi_y}.((b-m_{ij}) - m_\alpha)]$, where $\phi_y = \tau_\alpha + 1$

Which also equivalent to: $u(t) =$

$$\min([\overset{N}{\underset{r=1}{\oplus}}(A^{\phi_x})_{ir} - m_{ij}.m_\alpha].x_r(t-1), [\overset{N}{\underset{r=1}{\oplus}}(A^{\phi_y}).((b-m_{ij}) - m_\alpha)_{ir}].x_r(t-1))$$
$$= [\overset{N}{\underset{r=1}{\oplus}}(A^{\phi_x})_{ir} - m_{ij}.m_\alpha].x_r(t-1) \oplus [\overset{N}{\underset{r=1}{\oplus}}(A^{\phi_y}).((b-m_{ij}) - m_\alpha)_{ir}].x_r(t-1)$$

If the following conditions hold true:

$$m_\alpha \leq (b-m_{ij}).(A^k.B)_i \quad \text{for every } k = 0 \text{ to } \tau_\alpha \quad (10)$$

$$F_x \geq 0 \quad \text{and} \quad F_y \geq 0 \quad (11)$$

Proof.

The place p_{ij} is subject to mixed constraint of time and capacity of the forms (5) and (6).

Correspondingly, in the equation (5), if we substitute the counter function $x_i(t)$ by its expression that already given by (7), we will get the following inequation:

$$\left[\overset{N}{\underset{r=1}{\oplus}}(A^{\phi_x})_{ir} x_r(t-\phi_x)\right] \oplus \left[\overset{\phi_x-1}{\underset{k=0}{\oplus}}(A^k.B)_i.u(t-k)\right] \geq m_{ij} x_j(t-\tau_{ij}^{\max})$$

As a result, we deduce that the satisfaction of the mixed constraint represented by both equations (5) and (6) implies that of the following inequalities (12) and (13):

$$\overset{n}{\underset{r=1}{\oplus}}(A^{\phi_x})_{ir} x_r(t-\phi_x) \geq m_{ij} x_j(t-\tau_{ij}^{\max}) \quad (12)$$

$$\overset{\phi_x-1}{\underset{k=0}{\oplus}}(A^k.B)_i.u_x(t-k) \geq m_{ij} x_j(t-\tau_{ij}^{\max}) \quad (13)$$

From the inequality (8), if we substitute the term $x_j(t-\tau_{ij}^{\max})$ in both (12) and (13) the expression of these two inequalities become as follows:

$$\overset{N}{\underset{r=1}{\oplus}}(A^{\phi})_{ir} x_r(t-\tau_\alpha-\tau_{ij}^{\max}-1) \geq m_{ij}.m_\alpha.u_x(t-\tau_\alpha-\tau_{ij}^{\max}) \quad (12a)$$

$$\overset{\phi-1}{\underset{k=0}{\oplus}}(A^k.B)_i.u_x(t-k) \geq m_{ij}.m_\alpha.u_x(t-\tau_\alpha-\tau_{ij}^{\max}) \quad (13a)$$

By choosing $\phi_x = \tau_\alpha + \tau_{ij}^{\max} + 1$, in the (12a) and (13a) we notice that the second inequation (13a) is always verified since the counter function of the resource transition $u_x(t)$, is an increasing function and the terms $(A^k.B)_i$ are non-negative. After simplification, the inequation (12a) induces:

$$[\overset{N}{\underset{r=1}{\oplus}}(A^{\phi_x})_{ir} - m_{ij}.m_{\alpha_n}].x_r(t-1) \geq u_x(t)$$

Which is equivalent to $F_x x_r(t-1)$ part of the control that already introduced by (9).

Otherwise, to investigate the control $u_y(t)$ as a function of $x(t-1)$ we proceed similarly, the place p_{ij} is also subject to capacity constraint, we proceed by substituting equation (8) in (6), we get the following:

$$x_j(t) \leq (b-m_{ij})\left[\overset{N}{\underset{r=1}{\oplus}}(A^{\phi_y})_{ir} x_r(t-\phi_y)\right] \oplus \left[\overset{\phi_y-1}{\underset{k=0}{\oplus}}(A^k.B)_i.u_y(t-k)\right]$$

We assume that the satisfaction of the constraint (6) implies the satisfaction of both of the following inequalities:

$$x_j(t) \leq (b-m_{ij})\left[\overset{N}{\underset{r=1}{\oplus}}(A^{\phi_y})_{ir} x_r(t-\phi_y)\right] \quad (14)$$

$$x_j(t) \leq (b-m_{ij})\left[\overset{\phi_y-1}{\underset{k=0}{\oplus}}(A^k.B)_i.u_y(t-k)\right] \quad (15)$$

Now, considering (10), these two inequations are equivalent to:

$$m_\alpha.u_y(t-\tau_\alpha) \leq (b-m_{ij})\left[\overset{N}{\underset{r=1}{\oplus}}(A^{\phi_y})_{ir} x_r(t-\phi)\right] \quad (14a)$$

$$m_\alpha.u_y(t-\tau_\alpha) \leq (b-m_{ij})\left[\overset{\phi_y-1}{\underset{k=0}{\oplus}}(A^k.B)_i.u(t-k)\right] \quad (15a)$$

If we choose $\phi_y = \tau_\alpha + 1$, the inequation (14a) can be written as:

$$m_\alpha.u_y(t) \leq (b-m_{ij})\left[\overset{N}{\underset{r=1}{\oplus}}(A^{\phi_y})_{ir} x_r(t-1)\right]$$

It is equals to:

$$u_y(t) \leq ((b-m_{ij}) - m_\alpha)\left[\overset{N}{\underset{r=1}{\oplus}}(A^{\phi_y})_{ir} x_r(t-1)\right]$$

In this light, we admit that above inequality represents $F_y x_r(t-1)$ that establishes the desired control law in the case of capacity constraint on the same place, if the following conditions from (15. a) are satisfied.

$$((b-m_{ij}) - m_\alpha)\left[\overset{\phi_y-1}{\underset{k=0}{\oplus}}(A^k.B)_i.u(t-k)\right] \geq 0$$

$$m_\alpha \leq (b-m_{ij}).(A^k.B)_i$$

V. Application Case: Flexible Workshop

We introduce an application case that inlay within the framework of automated production systems. An example of a flexible workshop is suggested. Main Parts of this system are the dark room that is reserved for the chemical treatment task and the pick and place of two axis robot for the load/ unload task. Loaded products from an input area are picked by the robot arm and putted into the dark room for a dangerous chemical treatment during a time period parameterized depending on the piece's materials. After this period, the robot picks the piece and put it in the evacuation area. The Timed Event Graph of this application is given by Fig. 2.

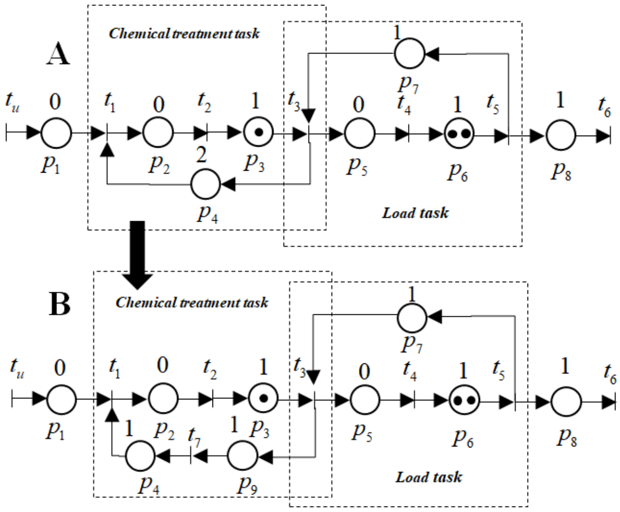

Figure 2. TEG of the flexible workshop

The task between t_1 and t_3 represents the chemical treatment. This process is time and capacity sensitive at a particular point which is illustrated by the place p_3. A suitable control must be applied in order to respect this mixed critical constraint which is essential for the proper functioning of the process. The modeling step consists on determining a Min-plus model meaning of linear equations such that the state space equation (3). As in the Fig. 2 the place is 2 time units we must use the expanded model given by **B**.

$$x(t)=\begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & e \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & e \\ \varepsilon & 1 & \varepsilon & \varepsilon & e & \varepsilon & \varepsilon \\ \varepsilon & 1 & \varepsilon & \varepsilon & e & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & 2 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & e & \varepsilon \\ \varepsilon & \varepsilon & e & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{bmatrix} x(t-1) \oplus \begin{bmatrix} e \\ e \\ \varepsilon \\ \varepsilon \\ \varepsilon \\ \varepsilon \\ \varepsilon \end{bmatrix} .u(t)$$

The place p_{32} have $\tau_{ij}^{\max}=\tau_{32}^{\max}$ which is the maximal time bound such that $\tau_{32}^{\max}=1$ time unit. In addition, as there exists a capacity bound such that $b=2,$ the mixed constraint is reduced to both following inequalities:

$$x_2(t) \le 1.x_3(t)$$

$$x_2(t) \ge 1.x_3(t-2)$$

The cumulated delay to be considered around the bath α is going from the resource transition t_u to the upstream transition of the constrained place t_2. It is given by: $\tau_\alpha=0$. $m_\alpha=0$ is the cumulated marking of the path α. In this case, we have $\phi_x=2$ and $\phi_y=1$. Then we could express the equation (7) according to $\phi_x=2$ as:

$$x_i(t)=\left[\bigoplus_{r=1}^{7}(A^2)_{ir}\, x_r(t-2)\right] \oplus \left[\bigoplus_{k=0}^{1}(A^k.B)_i.u(t-k)\right]$$

And according to $\phi_y=1$, we have:

$$x_i(t)=\left[\bigoplus_{r=1}^{7}(A^1)_{ir}\, x_r(t-1)\right]$$

Since this system's problematic corresponds to the problem solved through theorem 1, and after checking the following conditions:

$0 \le 1.(A^k.B)_3$ holding true for $k=0$ and $k=1$

and $F_x \ge 0$ and $F_y \ge 0$

The following control law guarantees the respect of the mixed constraints on the place and it is given like:

$$u(t)=\min(F_x x_r(t-1), F_y x_r(t-1))$$

$$u(t)=[\bigoplus_{r=1}^{N}(A^{\phi_x})_{ir}-m_{ij}.m_\alpha].x_r(t-1) \oplus [\bigoplus_{r=1}^{N}(A^{\phi_y}).((b-m_{ij})-m_\alpha)_{ir}].x_r(t-1)$$

$$=[\bigoplus_{r=1}^{7}(A^2)_{3r}\, x_r(t-1)] \oplus [\bigoplus_{r=1}^{7}(A^1).((2-1)-0)_{3r}].x_r(t-1)$$

$$=2.x_4(t-1) \oplus 1.x_7(t-1) \oplus 1.x_2(t-1) \oplus 1.x_5(t-1)$$

From the above control and the extended Timed Event Graph, we deduce that the firing time of the transition t_2 exceeds those of t_4, t_7 and t_5. Accordingly, the general control law that satisfies the mixed constraint on the place p_{32} is equals to: $u(t)=1.x_2(t-1)$.

The computed control law is represented by a delayed and marked place added to the extended TEG of Fig. 2.

Hence, we illustrate the controlled TEG in Fig. 3 below.

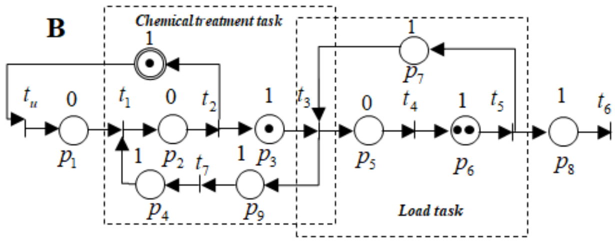

Figure 3. The controlled TEG of the constrained system

VI. Conculsion

TEG constitutes a major class within the paradigm of discrete event systems. This type of system seems to be more sensitive when the general process is exposed to mixed constraints related to time and capacity at once. Therefore the originality of the proposed problem resolution with Min-plus seminring through the existence of mixed constraints, provide an efficient solution; by checking the sufficient conditions, the resulting control laws satisfy these specifications. The graphical design of the controlled timed event graph witness the presence of added marked and delayed control places. Some interesting perspective for future works would be interesting to deal with performance evaluation for more complex TEGs cases, like CTEG.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

All authors contributed to the analysis of the problem of mixed constrained Discrete Event Systems. N. Ben Afia modeled analytically and graphically the flexible workshop application and wrote the paper; S. Amari and M. Hassani contributed to the calculation of the control synthesis and the verification of the mixed constraints; all authors had approved the version.

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