Comparison of the Balanced Truncation and Balanced Stochastic Truncation Algorithms in Model Order Reduction for IIR Digital Filters

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***Abstract*—In the field of system identification and modeling for dynamic systems, the increasing complexity and granularity of models often result in significantly higher computational costs and storage requirements, especially in real-time applications and optimization processes. To address this issue, model order reduction methods have been developed to produce reduced-order models with smaller dimensions while preserving the key dynamic characteristics of the original system. In this paper, two widely used model order reduction algorithms, Balanced Truncation (BT) and Balanced Stochastic Truncation (BST), are applied to reduce the order of high-order Infinite Impulse Response (IIR) digital filters. Analysis results reveal that BT outperforms BST in preserving the time-domain response and signal energy, whereas BST demonstrates superior performance in maintaining frequency-domain responses across the entire frequency range and preserving minimum-phase characteristics. Through comparisons of the reduction errors in terms of H_2 and H_∞-norms, the paper identifies that reducing the order from 30 to 11 and 15 achieves an optimal balance between accuracy and system complexity. These findings provide a basis for selecting appropriate algorithms in practical applications such as filter design, automatic control, and signal processing.**

***Keywords*—model order reduction, balanced truncation, balanced stochastic truncation, Infinite Impulse Response (IIR) digital filters, stability preservation, phase minimization**

I. INTRODUCTION

In the fields of system identification, modeling, analysis, and simulation, system complexity often arises from the increasing size and level of detail in models, aimed at achieving more accurate representations of system responses. However, this leads to significant challenges, such as increased computational costs, storage requirements, hardware expenses, and processing times—particularly in real-time systems or optimization problems that demand precise, continuous, and timely processing [1–4]. This necessitates model order reduction, a process of deriving reduced-order mathematical representations with fewer variables that can replace the original model while preserving the key dynamic characteristics or intrinsic properties of the initial system [5–9].

The significance of model order reduction lies in simplifying complex systems, enhancing efficiency in system design, analysis, and testing, and optimizing hardware resources to effectively meet the demands of practical applications [10–15]. One of the most widely used and effective methods for model order reduction is Balanced Truncation (BT). This algorithm is based on optimizing the system's dynamic energy after transforming it into a balanced Gramian space [16–25]. In this representation, system states are arranged in decreasing order of their energy contribution (Hankel singular values sorted from largest to smallest), allowing the elimination of less significant states (those with small Hankel singular values) without compromising system quality. The algorithm operates by computing the controllability and observability Gramians, followed by a transformation to bring the system into a positive definite balanced diagonal form. States with low energy are then truncated, yielding a reduced-order model that retains the essential characteristics of the original system.

In addition to Balanced Truncation, the Balanced Stochastic Truncation (BST) algorithm is also applied to systems operating in the frequency domain. BST is a model order reduction method designed to preserve critical properties of the original system, such as stability and minimum-phase characteristics [26–31]. A system is considered minimum-phase if all its zeros and poles are located on the left half of the complex plane. Minimum-phase characteristics have mathematical significance and reflect physical properties such as rapid and stable

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responses without inducing unwanted oscillations or excessive phase delays.

Infinite Impulse Response (IIR) filters are a type of digital filter commonly used in signal processing. These filters are often of high order to meet stringent signal filtering performance requirements, but this also increases complexity in their design and implementation [32, 33]. In this study, BT and BST algorithms are applied to reduce the order of a 30th-order IIR filter [34, 35] to lower orders, producing reduced models with improved computational efficiency while maintaining the response and essential characteristics of the original filter. A comparative analysis of the algorithms is conducted to ensure that the reduced-order models remain stable and reliable. The evaluation of the performance of these algorithms provides a basis for selecting the most suitable method, opening up potential applications in control systems.

Refs. [16–31] have indicated that many model order reduction methods exist, each with its own advantages and disadvantages, particularly in the context of IIR filters. This has created a gap for a direct comparison between the BT and BST algorithms. Although the improvements in BT and BST have proven effective in many applications, there remains a lack of comprehensive evaluation regarding their ability to preserve stability, frequency response, and the dynamic characteristics of the filter circuit after order reduction. Therefore, further in-depth research is required to establish a detailed basis for comparison, ultimately leading to optimal recommendations for selecting and applying the appropriate algorithm for each specific case.

This paper addresses the challenge of reducing the order of high-order IIR filters without compromising their essential signal characteristics. The main contributions of this study include:

- A detailed comparative analysis between the BT and BST algorithms based on H_2-norm and H_∞-norm, as well as time-domain and frequency-domain responses.
- Identification of optimal reduced orders (e.g., reducing from 30 to 11 and 15) that strike a balance between model accuracy and computational efficiency.
- Provision of practical guidelines for selecting the appropriate reduction method according to the specific requirements of control and signal processing applications.

II. Materials and Methods

Consider a standard Linear Time-Invariant dynamic system described by differential equations, state-space representation, or transfer functions as in Eq. (1) [22]:

$$\mathbf{G}(s):\begin{cases}\dot{\mathbf{x}}(t)=\mathbf{Ax}(t)+\mathbf{Bu}(t)\\ \mathbf{y}(t)=\mathbf{Cx}(t)+\mathbf{Du}(t)\end{cases}\Leftrightarrow \mathbf{G}(s):=\begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{bmatrix} \tag{1}$$

$$\mathbf{G}(s):=\mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}+\mathbf{D}$$

where $\mathbf{A}\in\mathrm{R}^{n\times m};\mathbf{B}\in\mathrm{R}^{n\times m};\mathbf{C}\in\mathrm{R}^{n\times m};\mathbf{D}\in\mathrm{R}^{n\times m}$ n is the system order, m is the number of inputs, and p is the number of outputs; **A**, **B**, **C**, **D** are the state matrix, input matrix, output matrix, and direct transmission matrix, respectively; **x**(t), **y**(t), **u**(t) represent the state vector, output vector, and input excitation vector, respectively.

The model order reduction (MOR) problem is defined as finding a reduced-order model ($r < n$) that approximates the original system, preserves certain physical properties, and improves computational efficiency.

Both BT and BST algorithms are foundational techniques in model reduction, offering robust and efficient solutions for simplifying complex systems while maintaining critical properties.

A. Model Order Reduction via Balanced Truncation Algorithm

Balanced Truncation (BT) is a widely used model reduction method in control theory and dynamic system modeling. This technique is mathematically grounded and optimizes the balance between maintaining the accuracy of the original system and reducing the model's size, thereby enhancing computational efficiency without significantly compromising the reliability of the outputs. BT is particularly beneficial for analyzing and controlling large-scale systems where direct handling is infeasible or computationally inefficient. The procedural steps for implementing the BT algorithm [16–25] are as follows:

- Step 1. Input Data Preparation: The original system is represented in the standard form Eq. (1).
- Step 2. Partition into Stable and Unstable Components: For unstable systems, the algorithm separates the original system into stable and unstable parts. Only the stable portion undergoes model reduction, ensuring that critical physical properties are preserved in the final result.
- Step 3. Solve Lyapunov Eqs. (2) and (3): Calculate the system's controllability and observability Gramians **P** and **Q** [22].

$$\mathbf{AP}+\mathbf{PA}^T=-\mathbf{BB}^T \tag{2}$$

$$\mathbf{A}^T\mathbf{Q}+\mathbf{QA}=-\mathbf{C}^T\mathbf{C} \tag{3}$$

These derivations follow the standard in system theory [17] and establish the basis for performing a balanced transformation, which allows for truncating the states with minimal Hankel singular values.

- Step 4. State-Space Transformation: Transform the state-space to balance the energy among states, identifying and ranking the states with minimal influence on the system dynamics.
- Step 5. Truncate Based on Hankel Singular Values: Discard the least significant states by retaining those corresponding to the largest Hankel singular values.
- Step 6. Formulate the Reduced-Order System: Derive the reduced-order model using the balanced and truncated matrices. The new system retains essential dynamic properties, including stability, with the error between the original and reduced systems bounded by the sum of the discarded singular values.

B. Model Order Reduction via Balanced Stochastic Truncation Algorithm

Balanced Stochastic Truncation (BST) is a model reduction technique that ensures the preservation of critical features such as frequency response, minimum-phase property, and stability. BST aims to derive a lower-order model from a high-order system while optimally retaining the statistical and energetic characteristics of the original system. The steps for implementing the BST algorithm [26–30] are as follows:

- Step 1. Declare the Initial System: Represent the original system in the standard form Eq. (1).
- Step 2. Compute Controllability and Observability Gramians: Solve the Lyapunov Eq. (4) and Riccati Eq. (5) to determine the Gramians **X** and **Y** [29].

$$\mathbf{AX} + \mathbf{XA}^{T} = -\mathbf{BB}^{T} \tag{4}$$

$$\boldsymbol{A}^T\boldsymbol{Y} + \boldsymbol{Y}\boldsymbol{A} = -(\boldsymbol{C} - \boldsymbol{B}_v^T \times \boldsymbol{Y})^T(\boldsymbol{D}\boldsymbol{D}^T)^{-1}(\boldsymbol{C} - \boldsymbol{B}_v^T \times \boldsymbol{Y}) \tag{5}$$

where: $\boldsymbol{B}_v = \boldsymbol{B} \times \boldsymbol{D}^T + \boldsymbol{X}\boldsymbol{C}^T$

- Step 3. Energy Balancing Transformation: Use a nonsingular transformation to balance the system's energy, converting it to a balanced form.
- Step 4. Order Reduction: Eliminate states with low energy contributions, typically identified via their corresponding singular values.
- Step 5. Performance Evaluation: Assess the new model's performance using criteria such as impulse response, step response, frequency spectrum comparison (e.g., Bode plots), or reduction errors based on dynamic norms.

III. Result and Discussion

Consider the IIR filter with the structure of Nth-order Lattice-Ladder shown in Fig. 1, and the transfer function G(s) of the IIR filter system of order $n = 30$ [29, 30] is as follows:

$$\mathbf{G}(s) = \frac{\mathbf{N}(s)}{\mathbf{D}(s)}$$

when:

$$\begin{aligned}\mathbf{N}(s) = {} & 0.0187s^{30} + 0.5044s^{29} + 6.574s^{28} + 55.13s^{27} + 334.2s^{26} + 1559s^{25} \\ & + 5824s^{24} + 1.788\times10^4 s^{23} + 4.597\times10^4 s^{22} + 1.003\times10^5 s^{21} \\ & + 1.878\times10^5 s^{20} + 3.035\times10^5 s^{19} + 4.261\times10^5 s^{18} + 5.216\times10^5 s^{17} \\ & + 5.578\times10^5 s^{16} + 5.219\times10^5 s^{15} + 4.269\times10^5 s^{14} + 3.05\times10^5 s^{13} \\ & + 1.897\times10^5 s^{12} + 1.023\times10^5 s^{11} + 4.753\times10^4 s^{10} + 1.888\times10^4 s^{9} \\ & + 6343s^{8} + 1777s^{7} + 407.4s^{6} + 74.54s^{5} + 10.52s^{4} + 1.088s^{3} \\ & + 0.076s^{2} + 0.0031s + 7.806\times10^{-18}\end{aligned}$$

$$\begin{aligned}\mathbf{D}(s) = {} & s^{30} + 25.96s^{29} + 325.5s^{28} + 2624s^{27} + 1.527\times10^4 s^{26} + 6.837\times10^4 s^{25} \\ & + 2.448\times10^5 s^{24} + 7.197\times10^5 s^{23} + 1.77\times10^6 s^{22} + 3.69\times10^6 s^{21} \\ & + 6.588\times10^6 s^{20} + 1.015\times10^7 s^{19} + 1.355\times10^7 s^{18} \\ & + 1.575\times10^7 s^{17} + 1.597\times10^7 s^{16} + 1.413\times10^7 s^{15} + 1.091\times10^7 s^{14} \\ & + 7.338\times10^6 s^{13} + 4.284\times10^6 s^{12} + 2.161\times10^6 s^{11} + 9.356\times10^5 s^{10} \\ & + 3.447\times10^5 s^{9} + 1.068\times10^5 s^{8} + 2.743\times10^4 s^{7} + 5719s^{6} + 941.3s^{5} \\ & + 117.6s^{4} + 10.49s^{3} + 0.6116s^{2} + 0.0222s + 0.0003\end{aligned}$$

The BT and BST algorithms are implemented in MATLAB software version 2023b to reduce the model order incrementally from the original order to order 1. For each reduced-order model, the model reduction errors are calculated using the H_2-norm and H_∞-norm, resulting in the plots shown in Figs. 2 and 3. These plots reveal that as the system order decreases from the original order (30th) to lower orders, both BT and BST methods exhibit an increase in error. This observation aligns with theoretical expectations since model order reduction eliminates certain dynamic components of the system, thereby introducing greater error in the reduced models. Notably, the errors between the original system and the reduced models for both H_2-norm and H_∞-norm are smaller for BT than for BST across all orders.

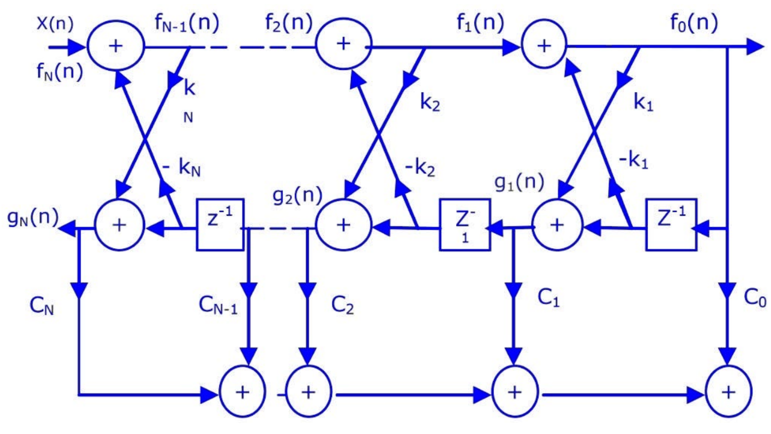

Fig. 1. Structure of Nth-order Lattice-Ladder IIR filter system [30].

Fig. 2 illustrates the H_2-norm error between the original system and the reduced-order models generated using BT and BST. This metric serves as a measure of the approximation accuracy between the reduced-order models and the original system in terms of signal energy. Smaller H_2-norm errors indicate higher accuracy of the reduced-order models, ensuring that critical signal energy characteristics are preserved in applications such as signal processing, automatic control, and communication systems.

From Fig. 2, the following observations are made:

- For H_2-norm errors with BT: The error decreases from 0.0429 at the lowest order (1) to a negligible value of 4.499×10^{-6} as the reduced order approaches the original order. The error decreases consistently with increasing r, without significant oscillations, demonstrating the stability of BT in model reduction.
- For H_2-norm errors with BST: The error also decreases from 0.0429 at the lowest order, but exhibits larger oscillations compared to BT. For instance, at order 3,

the error increases to 0.0526, significantly higher than BT's error of 0.0235.
- At higher orders, BST errors also reduce to values similar to those of BT but remain larger overall.
- BST generally produces larger errors than BT across most orders. Its reduced stability and accuracy at lower orders may complicate maintaining precision during model reduction.

Fig. 2. H_2-norm model reduction error for BT and BST.

Fig. 3 illustrates the H_∞-norm error between the original system and the reduced-order models for BT and BST. The H_∞-norm quantifies the maximum deviation between the original and reduced systems in the frequency domain. It provides an essential criterion for assessing system performance, particularly in applications where frequency response dictates system functionality, such as signal processing, robust control, and filter design. The H_∞-norm error is pivotal in evaluating the quality, reliability, and optimization of model reduction algorithms, ensuring that the reduced-order models retain critical dynamic characteristics of the original system.

From Fig. 3, the following observations are made:
- For BT: The H_∞-norm error decreases steadily as the order increases, starting from a maximum value of 0.2158 at order 1 and reducing to 6.146×10^{-6} near the original order. The error approaches zero, indicating that BT maintains better accuracy with minimal reduction in order.
- For BST: The H_∞-norm error also decreases with increasing order, but remains larger than BT at most orders. For example, at order 1, the BST error is 0.2158, identical to BT, but at order 2, it increases to 0.2486, significantly exceeding BT's error of 0.1629.
- At higher orders close to the original, BST errors also converge to zero but remain consistently larger than those of BT.
- BT demonstrates superior effectiveness compared to BST in H_∞-norm accuracy, particularly at medium and high orders. At lower orders (e.g., 1 to 5), BST exhibits more significant oscillations, suggesting that it may be less stable than BT under substantial order reductions.

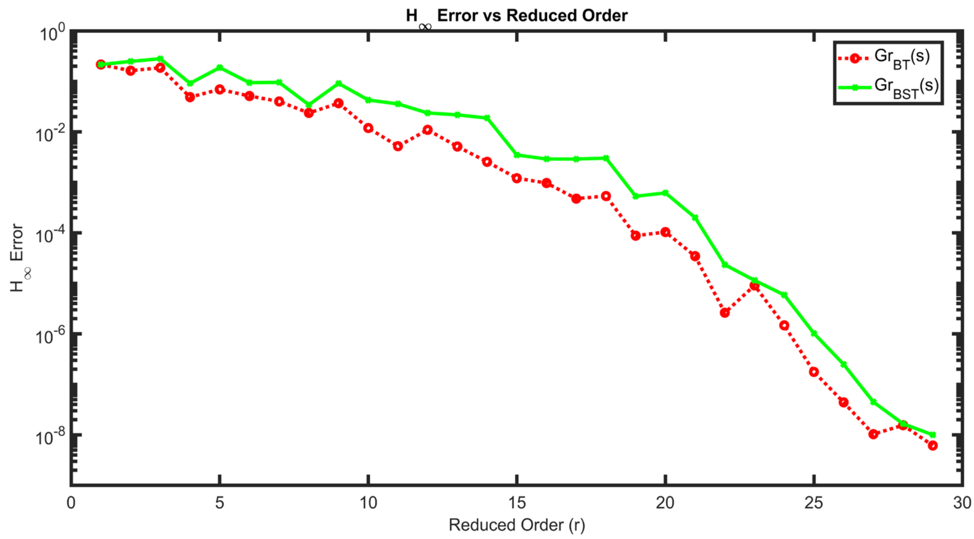

Fig. 3. H_∞-norm model reduction error for BT and BST.

Combining the two errors metrics and considering that lower order reductions should ideally yield minimal errors, it is observed that reductions to orders 11 and 15 yield substantially lower errors than other orders. At the same time, during the reduction process, it is necessary to consider the responses of both the reduced-order system and the original system in the time and frequency domains. After conducting numerous experiments on each reduced order, the authors analyzed, compared, and evaluated the results. Hence, the authors applied BT and BST to reduce the original 30th-order system to orders 11 and 15. The resulting H_2-norm and H_∞-norm errors for BT and BST are summarized in Table I, with impulse response and Bode diagrams for these reduced-order systems presented in Figs. 4 to 7.

TABLE I. ORDER REDUCTION ERROR ACCORDING TO H_∞-NORM AND H_2-NORM WHEN USING BT AND BST

Reduced Order (r)	Algorithm	H_∞-norm error	H_2-norm error
11	BT	0.005209	0.004064
	BST	0.035783	0.007988
15	BT	0.001209	0.000547
	BST	0.003493	0.000845

From Table I, it is evident that:

- The H_∞-norm error of G_BT(s), is significantly smaller than that of G_BST(s), indicating that BT offers better model approximation than BST.
- Similarly, the H_2-norm error of G_BT(s), is smaller than BST, demonstrating BT's superior capability in preserving the system's overall energy.

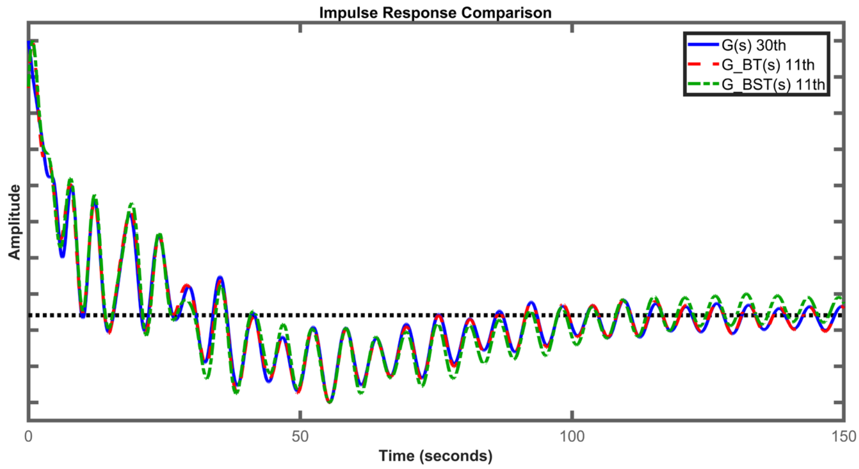

Fig. 4. Impulse response plots of the original system and the reduced 11th-order systems using BT and BST.

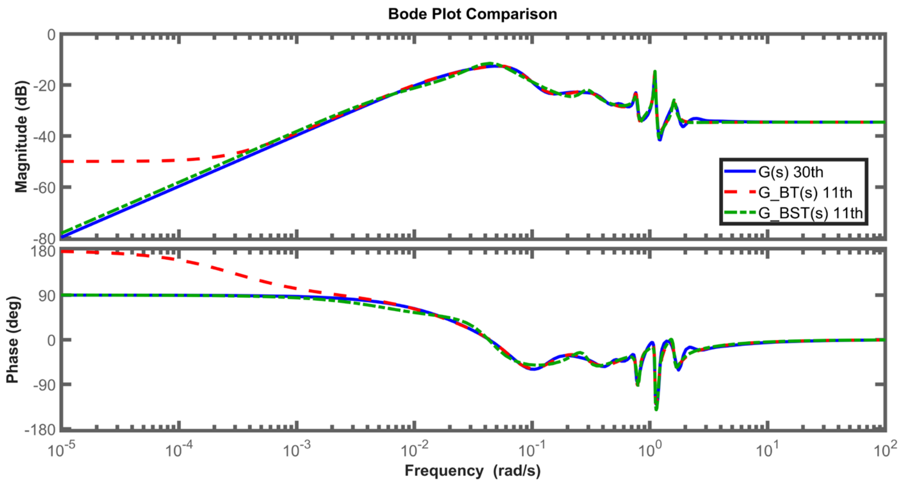

Fig. 5. Bode plots of the original system and the reduced 11th-order systems using BT and BST.

Figs. 4 and 5 further illustrate the impulse response and frequency response comparisons between the original and reduced-order systems for $r = 11$, showcasing the advantages of BT in time-domain and frequency-domain applications.

From the impulse response plots in Fig. 4, the following conclusions can be drawn:

- The impulse response of the original system, shown by the blue line (G(s), 30th order), exhibits damped oscillations over time with amplitudes gradually reducing to zero.
- The impulse response of the reduced system using BT (red line, G_BT(s), 11th order) and BST (green line, G_BST(s), 11th order): During the phase of large oscillations, both BT and BST align well with G(s). In the damping phase, G_BT(s) follows G(s) more closely than G_BST(s). This indicates that BT performs better in preserving the impulse response during the damping phase.

- Thus, BT provides better reduction quality in the time domain. Therefore, it is recommended to use the BT algorithm to reduce the original system to 11th order for time-domain applications. Alternatively, the BST algorithm can be used to reduce the system to 11th order for applications requiring accuracy during the phase of large oscillations.

From the frequency response plots in Fig. 5, the following observations can be made:

- The reduced system using BT (red line, G_BT(s), 11th order) aligns closely with the original system (blue line, G(s), 30th order) across most frequency ranges, with a slight deviation at low frequencies (approximately 10^{-5} rad/s to 10^{-3} rad/s).
- The reduced system using BST (green line, G_BST(s), 11th order) almost perfectly matches the original system across the entire frequency range.
- Thus, BST provides better reduction quality in the frequency domain. Therefore, it is recommended to use the BST algorithm to reduce the original system to 11th order for frequency-domain applications. Alternatively, the reduced system using BT can be utilized for applications in the medium to high-frequency ranges.

When reducing the original 30th-order system to a 15th-order system, the impulse and frequency responses of the systems are illustrated in Figs. 6 and 7.

From the time-domain response between the original system and the 15th-order system in Fig. 6, it can be observed that:

- Over the entire time domain, both datasets G_BT(s) and G_BST closely match G(s).
- It is feasible to replace the original 30th-order system with the 15th-order system using either the BT or BST algorithm in time-domain applications.

From the Bode plot comparison between the original system and the 15th-order system in Fig. 6, the following can be observed:

- In the medium- and high-frequency ranges, both BT and BST methods yield data that align with the original system G(s).
- In the low-frequency range, below 10^{-6} rad/s, the BT method shows greater deviations from the original system, whereas BST maintains data consistency with G(s).
- Consequently, the reduction quality of BST is superior to that of BT in the frequency domain. Therefore, the BST algorithm is recommended for reducing the original system to the 15th order and replacing the original system in frequency-domain applications across the entire frequency range. Alternatively, the 15th-order system obtained using BT can be utilized for applications involving frequencies above 10^{-3} rad/s.

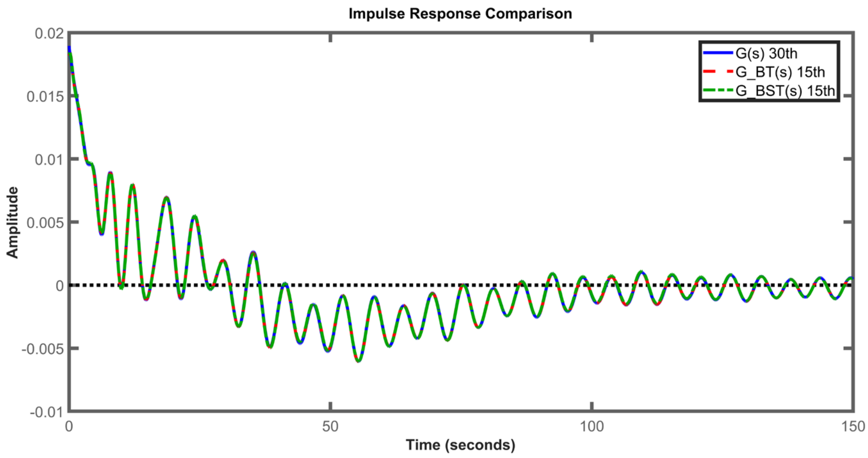

Fig. 6. Impulse response comparison between the original system and the reduced-order system (15th order) using BT and BST.

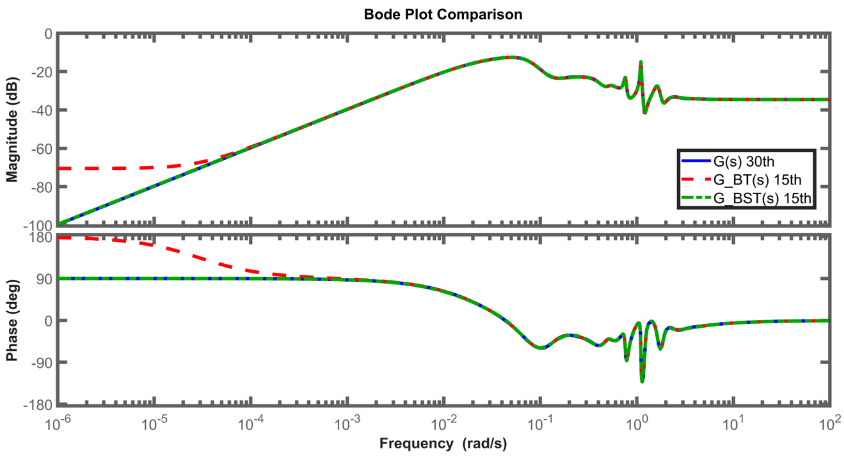

Fig. 7. Bode plot comparison between the original system and the reduced-order system (15th order) using BT and BST.

A. Overall Assessment

BT proves to be a more effective reduction method than BST when evaluated in the time domain, with superior stability and accuracy in terms of H_2-norm and H_∞-norm. Thus, BT is the preferred choice for applications requiring preservation of time-domain responses and signal energy.

On the other hand, BST demonstrates better performance than BT in the frequency domain. In addition to preserving stability in the reduced-order model, similar to BT, BST maintains minimum phase characteristics during reduction. Therefore, BST is a suitable choice for applications requiring frequency-domain response preservation across the entire frequency range, such as in filter design or robust control systems.

When reducing the original system to the 15th order, both BT and BST yield excellent results that align closely with the original system. This indicates that reducing the system from the 30th to the 15th order is optimal, striking a balance between system complexity and accuracy.

Both the BT and BST algorithms have a computational complexity of $O(n^3)$ for a system with a state matrix of size $n \times n$. However, in terms of actual constant factors, BST may be slightly more expensive due to solving a Riccati equation instead of two Lyapunov equations as in BT. Thus, both methods incur cubic computational costs, and the choice of algorithm may depend on the specific requirements of the problem and the characteristics of the system being reduced. In summary, the choice between BT and BST depends on the specific application requirements and the criteria for preserving the physical properties of the original system during reduction.

B. Discussion on Research and Development Directions

In the future, research can be advanced by conducting in-depth analyses of the statistical significance of error reduction through paired t-tests or Wilcoxon signed-rank tests to determine whether the BT algorithm outperforms BST in preserving the time-domain response, or vice versa. At the same time, comparisons with other model order reduction techniques—such as the Krylov Subspace method, Proper Orthogonal Decomposition (POD), and Balanced Singular Perturbation Approximation (BSPA)—should be carried out to evaluate the comprehensiveness and optimization potential of each method in the context of high-order systems. Moreover, analyzing the impact of BT and BST on hardware implementation on DSP or FPGA platforms should be emphasized to assess their ability to meet the requirements of real-time signal processing systems.

For specific IIR filter circuit models and high-order systems in general, the reduction process aimed at optimizing factors such as achieving the lowest possible order, minimizing reduction errors, ensuring an adequate time-domain response, and maintaining the desired frequency response requires testing and selecting among various approaches. Depending on the specific requirements and applications, simultaneously combining both BT and BST algorithms may yield better results than using each method individually; additionally, integrating supplementary optimization algorithms with a dedicated objective function is also a viable approach to achieving the desired reduction quality.

Furthermore, to enhance the visualization and interpretability of the reduction process, future studies should employ heatmaps or spectral plots to visualize the preserved frequency components in the model, while also comparing the phase responses between BT and BST to assess the extent of phase distortion. Error propagation analysis through error histograms and Nyquist plots should also be implemented to evaluate the robustness of the reduced-order models across different frequency bands. Finally, research directions should be expanded to include the integration of these algorithms into modern digital signal processing architectures, as well as exploring the feasibility of applying federated learning strategies for distributed model order reduction in large-scale adaptive filtering applications. In addition, investigating adaptive reduction techniques for real-time signal processing and multi-objective optimization strategies that balance model accuracy with computational cost will further open up extensive and effective application potentials in complex control and signal processing systems.

IV. Conclusion

This paper has studied and compared two popular model order reduction algorithms: Balanced Truncation (BT) and Balanced Stochastic Truncation (BST) in reducing the order of high-order IIR digital filters. The analysis results indicate that BT is more effective in preserving time-domain responses, stability, and signal energy, particularly in the time domain. In contrast, BST excels in maintaining frequency-domain responses across the entire frequency range and preserving minimum-phase characteristics, which is especially crucial in robust control applications and filter design. Both methods yield excellent results when reducing the order from 30th to 15th, with the order reduction error approaching zero, demonstrating that this reduction is optimal in balancing accuracy and system complexity. Depending on the specific requirements of the application, BT is the preferred choice for systems that require preservation of signal energy and stability, while BST is better suited for systems emphasizing frequency response and minimum-phase properties. These results not only confirm the effectiveness of the order reduction algorithms but also open up broad potential applications in signal processing, automatic control, and system optimization.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Conceptualization, Methodology, Formal analysis, Writing—original draft: Q.-X.H.; Software, Validation, Investigation, Resources, Data curation: V.-C.P., V-A.N. and D.-C.D.; Formal analysis, Writing—review & editing, Supervision, Project administration, H.-Q.N.; all authors have read and agreed to the published version of the manuscript.

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