Forecasting Volatility in Generalized Autoregressive Conditional Heteroscedastic (GARCH) Model with Outliers

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***Abstract*—This study aims to detect outliers and identify the best outlier detection technique and forecasting model for the financial time series data. Six outlier detection techniques and three forecasting models are compared to find the best technique and model using the daily returns data of the Pakistan Stock Exchange (PSX) 100 index from January 1996 to July 2020. The data is divided into two sections: the first estimate the model from January 1996 to December 2020, while the second produces one-day forecasts from January 2021 to July 2021. According to the research findings, the Mean Absolute Deviation (MADe) method of outlier identification outperforms the other outlier detection techniques in all three forecasting models with distinct loss functions. Furthermore, when comparing Generalized Autoregressive Conditional Heteroscedastic (GARCH) type models, Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH (1,1)) outperforms the other two forecasting models corresponding to the reported Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE). Therefore, the findings recommend that researchers use the MADe method to detect outliers and the EGARCH model as a forecasting model for financial time series data.**

***Keywords*—predictive analysis, outliers detection, Generalized Autoregressive Conditional Heteroscedastic (GARCH) models, time series, loss functions, economics growth**

I. Introduction

According to empirical and theoretical studies, a positive association between economic growth and the financial market has been identified in [1, 2]. With such a high impact of the financial market on economic development growth, predicting financial returns plays a significant role for investors in making decisions [3, 4]. Financial time series data frequently contain extreme values known as outliers. A value that deviates considerably from the rest of the observations in a data collection is referred to as an outlier. These outliers can be seen in the financial market on a periodic and random basis; they have a wide range of effects on data, and detecting these values in non-linear time series data is more crucial [5, 6]. Outliers in financial time series data can occur for various reasons, including significant financial difficulty, recession, development such as blockbuster goods, etc. Furthermore, the influence of these outlier returns (major events) significantly impacts the larger capitalization companies of the financial industry [7].

Outliers impair the accuracy of financial time series models, resulting in poor estimations of parameters calculated by these models. Shocks often occur in time series data, dramatically affecting the modelling identified in [8, 9]. This shows that data should be cleaned from outliers to avoid biased parameter estimation, volatility persistence, regularity conditions, heteroscedasticity, recognition of variance breaks, model misspecification, and poor forecasting as observed by Safari *et al.* [10], Charles and Darné [11]. Franses *et al.* [12] revealed that due to short patches of outliers, a false autoregressive conditional heteroscedastic (ARCH) effect may be caused through a gestured LM-ARCH test. There is also a possibility of not finding the actual ARCH effect in the presence of outliers in the data.

Further, Carnero *et al.* [13] concluded that only one outlier can vanish a true picture of asymmetry. The picture can be worse with spurious asymmetries if two giant outliers are present in the data. The same conclusion was observed for macroeconomic and financial variables. Several strategies for detecting and correcting outliers have been described in different research [14, 15]. Sakata and White [16] demonstrated that estimates using their proposed two-stage S-estimators and two-stage Hampel estimators for daily returns of the standard and poor's (S&P) 500 cash index series are unaffected by tiny

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outliers when compared to quasi-maximum likelihood based on conditional volatility models.

Ané *et al.* [17] demonstrated that their outlier detection and correction-based model produces more reliable forecasts than the Generalized Autoregressive Conditional Heteroscedastic (GARCH) model based on five Asian stock indices. Zhang *et al.* [18] investigated the effectiveness of their proposed diagnostics, curvature-based and slope-based, by empirically connecting GARCH modelling based on the daily returns of the New York stock exchange composite index. The study showed that both the diagnostics efficiently detected the cluster of additive shocks and are useful for measuring local impact, particularly in GARCH type models. Grané *et al.* [19] proposed an approach based on wavelet detection and correction of extreme values in accordance with the test developed by Bilen *et al.* [20]. The study revealed that in the presence of multiple isolated extreme values, the proposed technique performs well compared to other techniques to forecast volatility based on the daily stock index returns of the S&P 500, Dow Jones, and Financial Times Stock Exchange (FTSE) 100. Finally, Laurent *et al.* [21] introduced a semi-parametric robust technique to model volatility to detect and correct additive outliers in daily return data. It is concluded that basic GARCH models yield precise out-of-sample volatility forecasts when calculated using the suggested approach, filtered returns, followed by the GARCH model assessed from raw data.

TABLE I. A REVIEW OF COMPARATIVE STUDIES OF FORECASTING MODELS FOR FINANCIAL TIME SERIES DATA

Study	Methods/Models Compared	Conclusion
Sakata *et al.* [16]	the quasi-maximum likelihood, two-stage S-estimators, and two-stage Hampel estimators	two-stage S-estimators and two-stage Hampel estimators performed better
Kuhe [22]	GARCH (1,1), Glosten-Jagannathan-Runkle (GJR)-GARCH (1,1), and EGARCH (1,1)	EGARCH performed better
Ali [23]	EGARCH, GJR-GARCH, arithmetic power autoregressive conditional heteroskedasticity (APARCH), integrated generalized autoregressive conditional heteroskedasticity (IGARCH), nonlinear generalized autoregressive conditional heteroskedasticity (NGARCH), threshold generalized autoregressive conditional heteroskedasticity (TGARCH), and absolute value generalized autoregressive conditional heteroskedasticity (AVGARCH)	TGARCH performed better
Andersson and Haglund [24]	GARCH (1,1), GJR-GARCH (1,1), and EGARCH (1,1)	EGARCH performed better
Laurent *et al.* [21]	GARCH and generalized autoregressive score (GAS)	GARCH performed better
Ané *et al.* [17]	autoregressive (AR) (1)-GARCH (1,1), GARCH (1,1)	AR (1)-GARCH (1,1) performed better
Pasha *et al.* [25]	GARCH, GJR, EGARCH and APARCH	EGARCH performed better

This study aims to compare the forecasting performance of several GARCH type models for the returns of the PSX-100 index after outliers are eliminated. To this end, this research study first uses different conventional outlier detection techniques to detect outliers and then replace outliers through the average value of returns. Then, various GARCH type models are utilized following outlier replacement to select the optimal outlier detection strategy with accurate one-day-ahead forecasts. Finally, GARCH-type models are investigated to determine the best performing model that produces the overall least loss functions. As a result, the primary goal of this research is to compare conventional outlier detection techniques and compare GARCH type models to identify the better outlier detection technique and GARCH type model for financial time series data. Table I shows the comparative studies to model and forecast the financial time series.

The further research is organized into sections such that Section II goes through the six outlier identification approaches that were employed in this investigation, Section III evaluates three GARCH type models to model and make one-day-ahead forecasts; Section IV lays out the implementation of the tests; an empirical investigation based on the proposed methodology is discussed in Section V. Finally, the last section (Section VI) concludes by recommending future directions for research in light of this study.

II. CONVENTIONAL OUTLIER IDENTIFICATION METHODS

Numerous outlier detection procedures were reported for symmetric and skewed data with a univariate distribution. This research employs six outlier identification strategies to discover outliers in financial time series data. All six outlier identification techniques employ various measurements of central tendency (median, mean, etc.), dispersion measures, Inter-Quartile Range (IQR), and Standard Deviation (SD), etc. and have two fences to identify negative and positive outliers. These strategies, along with their mathematical structure, are detailed below.

A. 2SD Method

This is a basic approach for detecting outliers based on the normality assumption for the provided data. This approach is considered to be vulnerable to high values, Seo [26], and it is theoretically stated as follows:

$$[Lower\ Limit, Upper\ Limit] = [\overline{X} - 2\times Std, \overline{X} + 2\times Std] \quad (1)$$

Here $\overline{X}$ and Std represents mean and standard deviation of data under consideration. When a data set falls outside the above range, it is referred to as an outlier.

B. Tukey's Method

Tukey [27] developed this strategy, which is resistant to extreme values and uses the inter-quartile range as an indicator of dispersion and quartiles as a measure of central point to create a fence for the data under investigation. Mathematically, it is defined as,

$$[Lower\ Limit, Upper\ Limit] = [Q_1 - 1.5 \times IQR, Q_3 + 1.5 \times IQR] \quad (2)$$

Here Q_1 and Q_3 shows first and third of the given data set while IQR= $Q_3 - Q_1$ denotes the interquartile range of the data set. Any value recorded greater than the upper limit or lower than the lower limit is an outlier.

C. Modified Boxplot

Hubert and Vandervieren [28] coined the term "Modified Box-plot" to describe this boxplot approach. It uses Medcouple (MC) as a robust measure of skewness for skewed distributions rather than quartiles. When a data set, $y_1, y_2, ... y_n$, is individually selected from a continuous univariate distribution and sorted as $y_1 \le y_2 \le .... \le y_n$

$$MC(y_1, y_2, ..., y_n) = median \frac{(y_j - median_n) - (median_n - y_i)}{(y_j - y_i)} \quad (3)$$

Here "$median_n$" shows the median of the data under investigation and $y_i \le median \le y_j$, $y_i \ne y_j$. The VH boxplot interval is defined, as follows:

$$\begin{aligned} &[Lower\ Limit, Upper\ Limit] = [Q_1 - 1.5 \times \exp(-4MC) \\ &\times IQR, Q_3 + 1.5 \times \exp(3.5MC) \times IQR],\ if\ \ MC \le 0 \end{aligned} \quad (4)$$

$$\begin{aligned} &[Lower\ Limit, Upper\ Limit] = [\ Q_1 - 1.5 \times \exp(-3.5MC) \\ &\times IQR, Q_3 + 1.5 \times \exp(4MC) \times IQR],\ if\ \ MC \ge 0 \end{aligned} \quad (5)$$

Any value that falls outside the above-mentioned interval is recorded as an outlier.

D. Split Sample Skewness Based Boxplot (SSSBB)

Adil and Zaman [29] provided the most recent and robust outlier detection approach. This methodology follows the same structure as Tukey's method. The primary distinction is that it splits the entire data set into two equal parts before determining the quartiles and inter-quartile range for each portion [30] (i.e., to the left and right of the median). It is mathematically defined as,

$$[Lower\ Limit, Upper\ Limit] = [Q_{L1} - 1.5 \times IQR_L, Q_{R3} + 1.5 \times IQR_R] \quad (6)$$

where Q_{L1}, Q_{L3}, Q_{R1}, Q_{R3}, $IQR_L = Q_{L3} - Q_{L1}$, and $IQR_R = QR_3 - QR_1$ show 12.5th octile, 37.5th octile, 62.5th octile, 87.5th octile, inter-quartile range to the left of median, and inter-quartile to the right of median respectively. A value is outlier if it is higher than upper limit or lower than lower limit values.

E. Median Rule

Carling [31] created this approach, which is a robust measure of location parameters with a 50% cutoff. It splits the whole data set into two halves, one of which is greater than the median value and the other of which is smaller. Mathematically, this method is defined as:

$$[Lower\ Limit, Upper\ Limit] = [Q_2 - 2.3 \times IQR, Q_2 + 2.3 \times IQR] \quad (7)$$

Here $Q_2 = Median$, $Median = \frac{(y_m + y_{m+1})}{2}$ when m is even and when m is odd, respectively. Any observation in the data set that falls beyond the above-mentioned interval is considered an outlier.

F. MADe Method

This method is one of the fundamental approaches and is known to be unaffected by the occurrence of the data set's extreme values. This method is analogous to the SD method. However, this method uses median absolute deviation and median to make a fence. This method is expressed as

$$\begin{aligned} &[Lower\ Limit, Upper\ Limit] = [Median - 2 \times MADe, \\ &Median + 2 \times MADe] \end{aligned} \quad (8)$$

Here MADe=1.483×MAD

where $MAD = median\left(\left|x_i - median\left(x_i\right)\right|_{i=1,2,....,n}\right)$ represents an estimator showing spread in data and has an around 50% breakdown point similar to median. And due to this robust estimator, this method is not excessively affected by outliers. Thus, the interval is rarely overstated as compared to the SD method, though the distribution of a data set is skewed in the presence of few observations.

III. GARCH Type Models

Forecasting financial time series data volatility is getting increasingly popular. To date, in the literature, a significant variety of models for modelling and predicting financial time series data have been introduced. Among these models, GARCH type models get the greatest attention and are the most prevalent modelling and forecasting tools for financial time series data. The GARCH model is an econometric model that is used to study data gathered over time, with the variance error considered to be sequentially autocorrelated.

A. GARCH Model

Bollerslev [32] created the GARCH model, which is essentially a modified form of the ARCH model established by Engle [33]. GARCH model for the return's series (r_t) is defined as follows,

$$r_t = c + \rho_t, \qquad \rho_t \sim N(0, v_t^2) \quad (9)$$

and

$$\varpi_t^2 = \gamma_0 + \gamma_1 \rho_{t-1}^2 + \gamma_2 \varpi_{t-1}^2 \quad (10)$$

Eq. (10) is known as GARCH (1, 1), where represents the unexpected return or market shock. The other parameters must fulfil conditions $\gamma_0 > 0$, $\gamma_1 \ge 0$ and

$\gamma_2 \geq 0$ to assure that conditional variance will be positive. Further, the process is stationarity when the restriction $\gamma_0 + \gamma_1 < 1$ is fulfilled. Generally, GARCH model with p and q lag order is defined as,

$$\theta_t^2 = \gamma_0 + \sum_{i=1}^{p} \gamma_i \rho_{t-i}^2 + \sum_{j=1}^{q} \gamma_j \varpi_{t-j}^2 \tag{11}$$

B. GJR Model

Glosten *et al.* [34] developed their model to account for the influence of asymmetric leverage volatility. This model has the following specification of GJR-GARCH (1,1) for the conditional variance of model

$$\varpi_t^2 = \gamma_0 + \gamma_1 \rho_{t-1}^2 + \varphi I(\rho_{t-1} < 0)\rho_{t-1}^2 + \gamma_2 \varpi_{t-1}^2$$
$$\varpi_t^2 = \gamma_0 + (\gamma_1 + \varphi I(\rho_{t-1} < 0))\rho_{t-1}^2 + \gamma_2 \varpi_{t-1}^2 \tag{12}$$

where $I(\rho_{t-1} < 0) = 1$ if $\rho_{t-1} < 0$ otherwise. and 0 otherwise. For the positive volatility $\gamma_0 > 0$, $\gamma_1 > 0$, $\varphi \geqslant 0$, $\gamma_2 \geqslant 0$ and $\gamma_1 + \varphi \geqslant 0$. The given process is identified to be stationary if $\gamma_1 + \gamma_2 + (\varphi/2) < 1$ condition is satisfied. If $\varphi > 0$ then it means that asymmetry exists and negative and positive shocks of identical magnitude take diverse effects on conditional volatility. If the impulse $\gamma_1 + \varphi$ is identified to be of the negative shock, then asymmetry is identified, and this impulse has larger impact as compared to impact of impulse of positive shocks (γ_1). In this case, bad and positive news have differing effects on the model's conditional variance: bad news provides a huge impact of magnitude $\gamma_1 + \varphi$ while good news impacts last for a very short time. In the above model if $\varphi = 0$ then GJR-GARCH model becomes the GARCH model.

C. EGARCH Model

Nelson [35] proposed the exponential GARCH model Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH) to detect volatility leverage. This model may be represented as follows:

$$\ln(\theta_t^2) = \gamma_0 + \sum_{i=1}^{q} \left(\gamma_i \frac{\rho_{t-i}}{\theta_{t-i}} + \varphi \left| \frac{\rho_{t-i}}{\theta_{t-i}} \right| \right) + \sum_{j=1}^{p} \varpi_j \log\left(\theta_{t-j}^2\right) \tag{13}$$

This model demonstrates that the leverage effect is exponential, implying that forecasts for conditional variance are nonnegative. The hypothesis $\gamma_i < 0$ shows that the leverage effect is evident while if $\gamma_i \neq 0$ showing impact is asymmetric. The former condition ($\gamma_i < 0$) of shock generates more variability in the volatility than to the later ($\gamma_i \neq 0$). Fig. 1 shows the flowchart for the adopted methodology used in this study.

IV. Implementation of the Tests

To carry out this investigation, the following econometric approach is used:

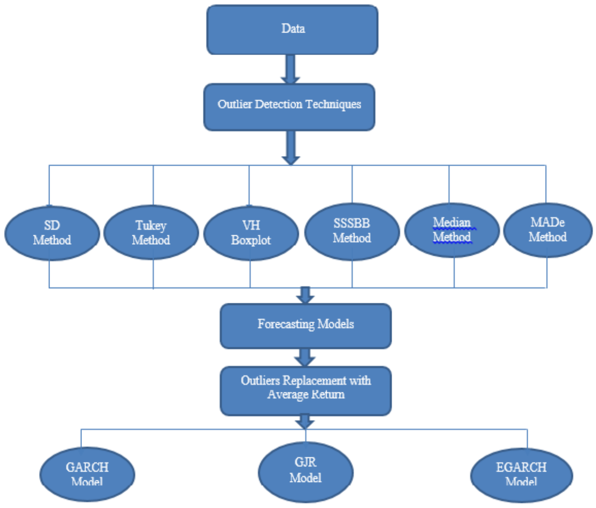

Figure 1. Proposed framework.

- To begin, the outlier identification approach is applied to the return series.
- Second, to create a new filter series, the number of outliers found by the outlier approach is replaced with the average return.
- Third, the previous two processes are repeated for all outlier detection approaches investigated in this work, yielding new filter series. Six new series are acquired in all.
- Fourth, the first GARCH type model is applied to all six filtered data to provide a one-step forecast.
- Fifth, the loss function values for each forecasting series are determined.
- Steps 4 and 5 are performed for the second and third GARCH type models.
- Finally, the top performance outlier detection approach and GARCH type model with the lowest loss function values are determined.

V. Outliers Treatment and Out-of-Sample Forecasts for the Pakistan Stock Exchange

We have taken daily returns data of PSX-100 index from January 1996 to July 2020 from www.tradingeconomics.com/pakistan/stock-market. The data is divided into two sections: the first covers the data range from January 1996 to December 2020 to estimate the model, while the second covers the range from January 2021 to July 2021 to make a one-day-ahead forecast. Fig. 2 shows a graphical depiction of the return series under consideration, with a black line dividing the model estimate and forecasting periods. Return is calculated $Y_t = (x_t - x_{t-1})/x_{t-1}$, here x_t and x_{t-1} denotes the current day and previous day stock price.

PSX is an emerging market known to be one of the top financial markets in Asia. PSX has achieved remarkable progress in its history, beginning with a limited presence of five listed businesses and a total amount of paid capital of Rs 37 million. There were 81 firms registered on the exchange in 1960, with a market valuation of Rs 1.8 billion. Still, there are presently 553 companies listed on the bourse, with a marketplace value of Rs 7.900 trillion. The listed firms are divided into 35 industry sectors or groups. Our main concern in the study is to view outliers in financial time series data and make forecasts after detecting and adjusting for them. Events such as the May 28, 1999 nuclear tests; General Ret. Pervaiz Musharraf taking over; the 9/11 attacks; the 2008 financial crisis; the assassination of Benazir Bhutto; the resignation of General Ret. Pervaiz Musharraf from the presidency and the election of Asif Zardari as the newly elected president; the tenure ending of Pakistan Muslim League Noon; Imran Khan as the newly elected prime minister; and the COVID-19 pandemic are considered to be the outlier events in financial time series of PSX-100.

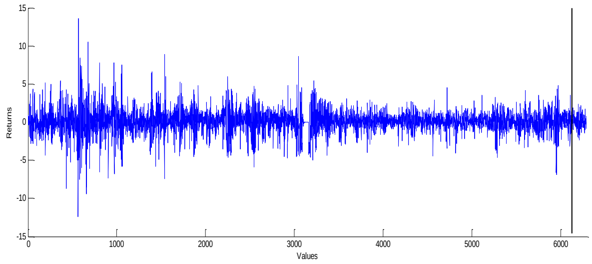

Figure 2. Observed daily returns for PSX-100 index for the period 1st January, 1996 to 30th July, 2021.

Table II shows the outliers' number detected by different techniques. The MADe procedure identified the maximum number of outliers (847) followed by the Tukey's method with 465 observations as outliers. This indicates that the MADe method detected major outliers and minor events, which disrupted the PSX-100 performance. This is because occasionally one anomaly (outlier) can disguise the other anomalies (outliers), and sometimes one anomaly (outlier) can expose an observation as an anomaly (outlier) when it is actually an inlier. Furthermore, for all techniques, the number of outliers on the right side of central points is larger than the outliers on the left side, indicating a positively skewed distribution for the under consideration return series. This also shows that positive news is more frequently affected by PSX than negative information. Similarly, Table III shows each technique's percentage number of outliers

TABLE II. NUMBER OF OUTLIERS IDENTIFIED BY OUTLIER DETECTION TECHNIQUES

	Left Outliers	*Right Outliers*	*Total Outliers*
SD Method	154	212	366
SSSBB	165	212	377
Median Method	131	197	328
VH Boxplot	163	299	462
MADe Method	392	455	847
Tukey Method	193	272	465

TABLE III. PERCENTAGE OF OUTLIERS IDENTIFIED BY OUTLIER DETECTION TECHNIQUES

	Left Outliers	*Right Outliers*	*Total Outliers*
SD Method	2.4	3.4	5.8
SSSBB	2.6	3.4	6.0
Median Method	2.1	3.1	5.2
VH Boxplot	2.6	4.8	7.3
MADe Method	6.2	7.2	13.5
Tukey Method	3.1	4.3	7.4

TABLE IV. MEAN ABSOLUTE ERROR: DESCRIPTIVE STATISTICS FOR DIFFERENT COMBINATIONS OF OUTLIER DETECTION TECHNIQUES AND FORECASTING MODELS

Outlier Detection Techniques	MAE (Forecasting Models)		
	GARCH(1,1)	***EGARCH(1,1)***	*GJR(1,1)*
With outliers	**0.519915**	**0.504979**	**0.52728**
SD Method	0.303514	**0.296027**	0.317051
Median Method	0.315387	**0.310751**	0.331003
MADe Method	**0.08749**	**0.083056**	**0.105053**
Tukey Method	0.295266	**0.28491**	0.320874
VH Boxplot	0.322555	**0.312523**	0.351099
SSSBB	0.300098	**0.28973**	0.311441

The forecasting results of different outlier detection techniques corresponding to each considered forecasting model for returns are given in Table IV and Table V. The only difference between the tables is the loss function, which is different for both tables. Table IV reports MAE results when outlier detection techniques correspond to forecasting models considered on the return series. It is evident that the MADe method with minimum values of MAE, compared to other outlier techniques, provides good forecasts corresponding to all three forecasting models. On the other hand, it is evident that forecasts of return series with outliers provide the worst performance, having the maximum MAE for all forecasting models compared to all outlier detection techniques. This explains and justifies the results obtained in [11, 13]. Similarly, comparing the forecast performance of GARCH type models, the EGARCH (1,1) model with the least MAE performs better as compared to GARCH (1,1) and GJR (1,1) models.

TABLE V. MEAN ABSOLUTE PERCENTAGE ERROR: DESCRIPTIVE STATISTICS FOR DIFFERENT COMBINATION OF OUTLIER DETECTION TECHNIQUES AND FORECASTING MODELS

Outlier Detection Techniques	MAPE (Forecasting Models)		
	GARCH(1,1)	***EGARCH(1,1)***	*GJR(1,1)*
With outliers	58.96145	**48.14709**	58.43016
SD Method	38.67869	**33.0133**	39.99078
Median Method	40.12212	**34.27311**	41.7954
MADe Method	**15.04214**	**13.5973**	**17.7419**
Tukey Method	38.56718	**33.54817**	41.84462
VH Boxplot	39.52747	**35.29837**	43.43097
SSSBB	38.19992	**32.31972**	39.23744

Table V shows the result of MAPE obtained after applying different outlier detection techniques corresponding to different forecasting models to the return series. The results are consistent with Table IV, in which the MADe method with the lowest MAPE identified the best performer outlier detection technique in terms of forecasting models when compared to all other techniques.

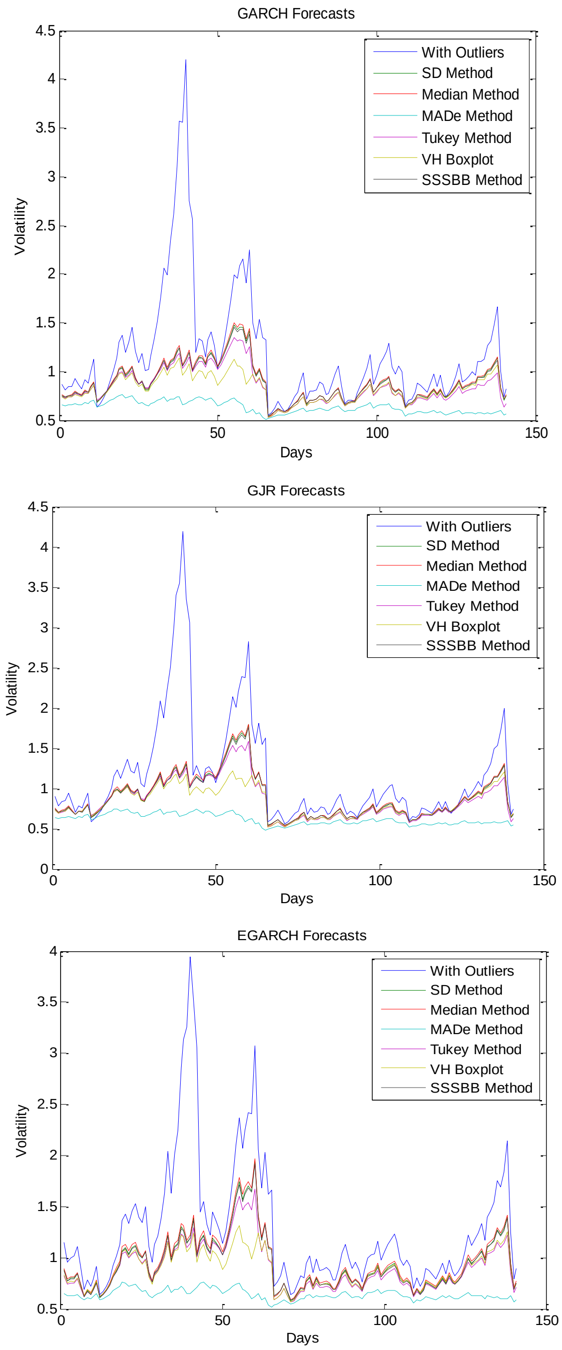

Figure 3. One day ahead out of sample forecast of GARCH (1,1), EGARCH (1,1) and GJR (1,1) models with different outlier detection methods from 1st Jan 2021 to 30th Jul 2021.

On the other hand, return series with outliers produce poor performance, with maximum MAPE corresponding to all three forecasting models. This indicates the importance of outliers' detection and replacement for the financial time series data. Finally, concerning the results of forecasting models, EGARCH (1,1) with the least MAPE for all outlier detection techniques is identified to be the better performing forecasting model as compared to the performance of GARCH (1,1) and GJR (1,1) models.

The overall forecasts of all three GARCH type models without and with treatment of outliers through six outlier detection techniques are further depicted in Fig. 3 showing the much volatile forecasts pattern when predictions made in the presence of outliers as compared to forecasts pattern after treatment of outliers. While almost stable forecasts behavior fluctuates between 0.5 to 1, forecasts are obtained from all three forecasting models with treatment of outlier from the MADe method. Thus, this outlier detection method outperforms other methods for our investigated data.

It is worth noting that the forecasting ability of the models we investigated to model financial time series data is very competitive with the findings reported in the literature. For example, forecasting from series with outlies produces poor performance compared to all considered investigated forecasting models that favour studies of Safari *et al.* [10], Charles and Darné [11].

Our findings show the optimal forecast supremacy of GRACH type models compared to other models reported in the literature with respect to different loss functions.

Moreover, as far as PSX is concerned, our results parallel the findings of Pasha *et al.* [25] showing optimal forecasts of the EGARCH (1,1) model, whether in long-run or short-run time series [36, 37].

VI. Conclusion

This study revisited the problem one day ahead of the return forecast in the presence and without outliers in financial time series data, which is considered volatile data. As a result, detecting extreme (outlier) values plays an important role in identifying less and more volatile time periods and providing better forecasts after outlier replacement. This study evaluated the performance of six outlier detection techniques and three GARCH type models with respect to obtaining forecasts. First, each outlier detection technique applied to the return series and the identification of outliers have been captured. Then replacement of outliers is done with average series returns of the series under consideration to produce a filter series. Second, different GARCH type models have been utilized with and without filter series to make the forecasts. Data is divided into two parts; a model is estimated using the first part of the data while the second part is used to produce one-day-ahead forecasts. Finally, the forecasting performance of different outlier detection techniques and models has been evaluated on the basis of loss functions. It is observed that when outliers are detected using the MADe method and forecasts are made using different forecast models, this method produces good forecasts compared to other outlier detection techniques. Similarly, the EGARCH (1,1) model outperforms GARCH (1,1) and GJR (1,1) models with respect to forecast performance, having a minimum

MAPE and MAE corresponding to all considered outlier detection techniques.

The study's empirical findings imply that, among all other approaches, the MADe method should be utilized to identify outliers in financial time series data owing to its real-time identification of outliers. While in the comparison of considered forecasting models, the EGARCH (1,1) model with the least MAE and MAPE should be regarded for forecasting purposes when outliers in financial time series data are present and filtered. As this study is limited only to PSX-100 index data, other stock market analysis is recommended to evaluate outlier detection methods and forecasting models further. Also, this study is limited to only three forecasting models. Therefore, a comprehensive empirical study must consider and compare the maximum number of forecasting models to identify the best model.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

SA contributed in Conceptualization, Data curation, Investigation, Methodology; writing; TS involved in Investigation, Methodology, Resources, Writing – review & editing; SAB contributed in Data curation, Investigation, Resources, MI contributions are in Software, Writing – original draft, Writing – review & editing; ARK involved in Conceptualization, Data curation, Investigation, Methodology, Resources, Software, review, editing original draft. All authors had approved the final version.

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