The Comparison of Techniques for Estimating and Measuring the Movement Time of the Right Thumb in a Curve on a Smartphone Based on Fitts’s Law

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Abstract—The goal of this research is to compare the techniques that estimate and measure the movement time of the right thumb in a curved path on a smartphone according to Fitts’s law. A coordinate randomizing tool was created in this study to randomize the coordinates from the starting point to the target button. The tool was programmed to make an angle of 45-degrees between the starting point and the target button. The experiment was then conducted on 120 Thai university students and the results were analyzed using Mean Absolute Error (MAE) of the estimated or standard movement time. The findings indicated that the technique of measuring distance between buttons using arc length and vertical width has the lowest MAE. The result of this study can be used to improve object placement in user interface and user experience designs.

Index Terms—mobile interactions, human-computer interaction, Fitts’s law, button size, gesture, ergonomics

I. INTRODUCTION

The analysis and design of human-computer interactions, particularly those involving humans and a smartphone has become increasingly complicated [1], [2]. The factors that influence human-smartphone interaction differ from those that influence humans and other devices since a smartphone screen is smaller and vertically-oriented, unlike the horizontally-oriented screens of Personal Computers (PC) and laptops.

Fitts’s law is used to assess human-computer interactions and to estimate the movement period [1]-[11] and index of difficulty. However, Fitts’s law is generally suitable for measuring and estimating movement time between objects on the same axis or in one dimension. In other words, the measuring axis can either be horizontal or vertical, and it cannot be employed to measure both axes at the same time.

Therefore, the estimation of the movement time and the index of difficulty of human-smartphone interactions is limited.

In addition, the curved movement path of the fingers and the ergonomics must be considered while designing the interaction [12]-[14] since smartphones are typically held with one hand.

A human thumb plays the most crucial role in a hand movement when using a smartphone because it performs the basic hand movement activities. The relationship between the thumb muscle activity and thumb usage on a touch screen (a right-hand motion) in humans of various age ranges has been investigated, and it has been discovered that the size of the touch button impacts the user interface design [15] as well as basic motions, including tapping, dragging, pinching, and other basic interactive elements [16].

This study employed Fitts law for experimentation and then compared the techniques used for estimating and measuring the movement durations of the right thumb in a curved path. The Euclidian distance and arc length techniques used to measure the size of the buttons on a website were compared to determine the best target size for the right thumb. The optimal target size affects the design of the buttons employed by the right thumb as it moves in a curved path. It also aids in the improvement of button designs ensuring they have adhered to user experience and ergonomic principles. It can also be used as a guideline for developing user-friendly applications.

II. LITERATURE REVIEW

A. Fitts’s Law

Fitts’s law was first proposed in 1954 by the American psychologist, Paul M. Fitts. It involves the measurement of movement time from a position to a target [17].

Fitts’s law has been applied to website design to estimate the movement time and the index of difficulty of designed user interfaces to optimize the distance between buttons or hyperlinks, as well as their sizes, and to improve user convenience and precision when accessing the information on a website [11], [18]-[22].

The measurement of movement time is based on the hypothesis that the harder the movement, the more time is needed to complete it. The index of difficulty is calculated from the relationship of the distance between the starting point and center and the width of the target.

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Fitts’s law can be expressed in a mathematical formula [3] as follows:

\[ MT = a + b[ID] \]

and

\[ ID = \log_2 \left( \frac{D}{W} + 1 \right) \]

where

- \( MT \) is the movement time.
- \( D \) is the distance between the starting point and the center of the target.
- \( W \) denotes the width of the target.
- \( a \) signifies the regression coefficient, which is a delay caused by the equipment or any delay created by an event, which may influence the calculation of the movement time.
- \( b \) is the index of difficulty, with bits as its unit.
- \( ID \) is the index of difficulty of 1 bit.

Remark: If there is no time-related estimation of the equipment or event, the regression coefficient of constant \( a \) is 0 and that of constant \( b \) is 1.

Fig. 1 shows an illustration of a website with ‘OK’ and ‘cancel’ buttons placed horizontally beside each other (0-degree axis). The size of both buttons is 200px and the distance between the center of the first button and that of the second button is 250px.

![Figure 1. The measurement of the distance for Fitts’s law on a 0- and 90-degree axis.](image)

Fig. 2 indicates the calculation of the movement time between the two buttons where a PC has an estimated index of difficulty of 1 bit (which takes 100ms).

![Figure 2. An example of the ratio and distance between the two buttons.](image)

Substitution

\[ D = 250 \text{ px} \]
\[ W = 200 \text{ px} \]
\[ b = 100 \text{ ms} \]

Therefore

\[ MT = 100 \left[ \log_2 \frac{250}{250} + 1 \right] \]
\[ = 100 \times 1.169 \]
\[ = 116.99 \text{ ms}. \]

According to Fig. 2, when Fitts’s law equation was substituted with values, the movement time from the ‘OK’ to the ‘cancel’ buttons is 116.99 ms.

B. The Model Human Processor

When humans interact with a computer device or a smartphone, they perceive and process according to the Model Human Processor (MHP). MHP is a model that estimates the duration for processing and responding to a machine. MHP was first developed by Stuart K. Card, Thomas P. Moran, and Allen Newell in 1983 [23].

The MHP comprises three parameters; 1) perception processor, 2) cognitive processor, and 3) motor processor. Table I lists the cycle time of human processors from the beginning to the end.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perceptual processor cycle time</td>
<td>100 ms</td>
<td>50–200 ms</td>
</tr>
<tr>
<td>Cognitive processor cycle time</td>
<td>70 ms</td>
<td>25–170 ms</td>
</tr>
<tr>
<td>Motor processor cycle time</td>
<td>70 ms</td>
<td>30–100 ms</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>240 ms</strong></td>
<td><strong>-</strong></td>
</tr>
</tbody>
</table>

Table I displays the processor cycle time by parameter, which can be used as the coefficient to compute the estimated time for finger movement on smartphones from the starting point to the target. In this study, the one-bit index of difficulty took 240ms.

C. The Euclidean Distance

The Euclidian distance, which is based on the Pythagorean theorem, is the measurement of the straight line distance between two points. The Euclidian distance between two points is the length of a straight line in Cartesian coordinates and is calculated with the following formula [24]:

\[ d(p, q) = \sqrt{ \sum_{i=1}^{n} (p_i - q_i)^2 } \]

where

- \( d \) is the distance between point \( p \) and point \( q \).
- \( p \) is the coordinate of the first point, consisting of \( p_1 \) substituting for the coordinate on the x-axis and \( p_2 \) substituting for the coordinate on the y-axis.
- \( q \) signifies the coordinate of the second point, consisting of \( q_1 \) substituting for the coordinate on the x-axis and \( q_2 \) substituting for the coordinate on the y-axis.
- \( i \) denotes the number of dimensions.
D. The Arc Length

The arc length is the measurement of the distance between two points along a section of a curve. It is calculated using the following formula [25], [26]:

\[
\text{Arc Length} = 2\pi r \left(\frac{C}{360}\right)
\]

where

- \text{Arc Length} is the distance between the starting point and the target along a section of a curve.
- \( r \) denotes the radius of the circle.
- \( C \) is the angle at which the arc subtends at the center of the circle in degrees.

E. The Mean Absolute Error (MAE)

In this research, the MAE was calculated for comparison, and to determine the best approach for measuring the index of difficulty as well as the movement time of the right thumb in a curved path on a smartphone. The most suitable process should have the lowest MAE. The MAE is calculated using the following formula [27]:

\[
\text{MAE} = \frac{1}{n} \sum_{i=1}^{n} |y_i - y|
\]

where

- \( \text{MAE} \) signifies the mean absolute error.
- \( n \) is the number of vectors to measure the error.
- \( i \) denotes the pair order of the error on the vector.
- \( y \) is the value of the standard variable obtained from the estimate.
- \( y_i \) signifies the value of the base case from the actual measurement.

III. RESEARCH METHODOLOGY

This research compared the Euclidian distance and arc length techniques utilized to estimate and measure the movement durations of the right thumb in a curved path on a smartphone according to Fitts’s law. To fulfill the research goals, the research comprised three stages as follows:

A. The Development of a Coordinate Randomizing Tool for Recording the Movement Times of the Right Thumb

In this stage, a coordinate randomizing tool was designed and programmed by the researcher using HyperText Markup Language (HTML), Document Object Modelling, and JavaScript Language. The coordinates of the start and target buttons were randomized using the tool. Both buttons were 116px tall by 190px wide. They were located on a smartphone with a screen size of 960px by 1707px. The tool was regulated so that the starting point always formed a 45-degree angle to the target button. Therefore, the buttons were never on the same plane, both horizontally or vertically, as indicated in Fig. 3.

B. The Measurement and Comparison of the Index of Difficulty of the Euclidian Distance and Arc Length Measuring Techniques

This stage was divided into two phases as follows:

Phase 1: comprised the measurement of the index of difficulty using the Euclidian distance and arc length measurements.

In this phase, the researcher developed a randomizing tool for randomly determining the coordinates of the starting point and a target for the button size on a website. The tool randomized the coordinates of the starting point and a target, which were not on the same plane, and made a 45-degree angle. The coordinates were set apart from the right edge of the screen at five intervals: 200px, 300px, 400px, 500px and 600px. Four measuring techniques, such as the Euclidian 1, Euclidian 2, arc length 1, and arc length 2 were employed. The cycle time of the human processors was set at 240 ms as shown in Table II.

<table>
<thead>
<tr>
<th>Method</th>
<th>Type of distance measurement between the starting point and the target (D)</th>
<th>Button width (W)</th>
<th>Cycle time of the human processor (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euclidian distance 1</td>
<td>Euclidian</td>
<td>Vertical width W = 116px</td>
<td>240 ms</td>
</tr>
<tr>
<td>Euclidian distance 2</td>
<td>Euclidian</td>
<td>Width from Euclidian distance W' = 164.05px</td>
<td>240 ms</td>
</tr>
<tr>
<td>Arc length 1</td>
<td>Arc Length</td>
<td>Vertical width W = 116px</td>
<td>240 ms</td>
</tr>
<tr>
<td>Arc length 2</td>
<td>Arc Length</td>
<td>Width from Euclidian distance W' = 222.61px</td>
<td>240 ms</td>
</tr>
</tbody>
</table>

TABLE II. THE EUCLIDIAN DISTANCE AND ARC LENGTH TECHNIQUE WERE USED IN THIS RESEARCH TO MEASURE THE INDEX OF DIFFICULTY
According to Table II, the vertical width was set to 116px for Euclidian 1, whereas the width from the Euclidian distance was 222.61px for Euclidian 2. The line for measuring the width made a 45-degree angle with the top of the button and button borders. The cycle time of the human processor (b) was set to 240 ms as shown in Fig. 4.

![Figure 4. The measurement of the button width for the Euclidian distance technique.](image)

As summarized in Table II, the vertical width is set to 116px for arc length 1 whereas the width from the Euclidian distance is 222.61px for arc length 2. The line used to measure the width made a 45-degree angle with the top and borders of the button. The cycle time of the human processor (b) was set to 240 ms as shown in Fig. 5.

![Figure 5. The measurement of the button width for the arc length method.](image)

Fig. 6 illustrates the measurement of the distance from the starting point to the determined target point using the Euclidian distance 1 and 2 methods to identify the starting point and the target for the button size on a smartphone. The tool randomized the coordinates of the starting point and the target which were not on the same plane to make a 45-degree angle. The randomized coordinates were set aside from the right screen edge at five intervals: 200px, 300px, 400px, 500px and 600px as shown in Fig. 7.

![Figure 6. The measurement of the button width for the Euclidian distance 1 and 2 methods.](image)

Phase 2: Testing with the Sample Group: The researcher employed a random sampling method to select a sample group of 120 university students aged 18–22 years. They were each asked to use the developed tool 30 times. Their coordinates and actual movement times were recorded.

C. Measuring the Errors

In this phase, the researcher correlated the average MAE of the movement times of the right thumb moving in a curved path on a smartphone obtained from the estimates or standard measurements ($MT_{est}$) with the actual movement times ($MT_{act}$) recorded during the experiment.

IV. RESEARCH FINDINGS

The findings were in accordance with the measured index of difficulty and the estimated or standard movement time ($MT_{est}$) at five intervals: 200px, 300px, 400px, 500px and 600px. Four measuring techniques, such as the Euclidian distance 1 and 2, and the arc length measuring methods 1 and 2 were used. The index of difficulty and estimated or standard movement times are displayed in Table III. The estimated or standard movement times were then compared with the actual movement times ($MT_{act}$) from the experiments. The results are shown in Table IV.
The arc length 1 measuring method had the lowest index of difficulty in all devices.

The objectives of the experiment were to compare the measuring techniques and their estimated movement durations for the right thumb moving in a curved path on a smartphone according to Fitts’ law to determine the most suitable target size for one-handed operations. The findings indicated that the most appropriate target should be measured by the arc length 1 method, which measures the arc length of the circle from the starting point to the target, made at a 90-degree angle with the measurement center. The vertical width of the button was 116px, and the coordinate was set at 200px, 300px, 400px, 500px and 600px from the right edge of the screen.

Based on Fitts’ law, the arc length 1 measuring approach provided estimated movement times for the right thumb moving in a curved path on a smartphone, which was close to the actual movement times. This is in line with the finding of a study [5] which stated that mean movement times are significantly linked to the target’s index of difficulty in all devices. Furthermore, there have been studies examining the optimal target size for a single thumb movement while using a mobile device that depends on the distance to the direction of the target [11], users’ tapping behavior during one-handed interaction [28], and other factors, such as age, thumb length, screen size [29], and the angle range of the thumb movement direction affecting the thumb movement [30]. Further studies for the consideration of the next phase of the experiment may include interactions with different types of touch screens, multi-touch movement time estimation techniques, or the estimation of movement times of right-hand and left-hand fingers. The study and development of estimating and measuring techniques will be essential in understanding the size of the objects with which users interact on touch screens of different types and sizes for the development of software and hardware that require input from human finger interaction.

\[ MT_{Arc\ Length1} = MHP(ID) \]
\[ ID = \log_2 \left( \frac{Arc}{W} + 1 \right) \]
\[ Arc = 2\pi r \left( \frac{90}{360} \right) \]

where

- \( MT_{Arc\ Length1} \) is the arc length 1 measuring method used to measure the movement time of the right thumb from the starting point to the target.
- \( MHP \) denotes the perceptual cycle time of the human model, which was set to 240 ms.
- \( ID \) is the index of difficulty.
- \( Arc \) signifies the measurement of distance on an arc.

The mean absolute error (MAE) was calculated for each measuring method.

\[
\text{MAE} = \frac{1}{n} \sum_{i=1}^{n} |\text{Estimated MT} - \text{Actual MT}| \]

The result in Table IV, can be summarized as shown in Fig. 8.

![Figure 8. The average MAEs for each measuring method.](image)

According to Tables III and IV, the results revealed that the arc length 1 measuring method had the lowest MAE compared with the other measuring techniques. The formula for the calculation is as follows:

- \( W \) signifies the vertical width of the button, which is 116px.
- \( r \) signifies the radius of the circle.

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V. CONCLUSION

The experiments in this research compared the estimates and measurements of the movement times of the right thumb in a curved path on a smartphone based on Fitts’s law. The estimates and measurements correlated were obtained using the Euclidian distance 1 and 2 methods and the arc length 1 and 2 methods. The MAEs of the estimated MTs were compared with the actual MTs obtained from each measuring method. The arc length 1 technique was discovered to offer the estimated time close to the actual movement time. This approach could be employed to estimate and measure the movement times of other user interfaces, including those designed for websites, mobile sites, smartphone applications, device control screens, and others.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

This research work was mainly conducted by Jarunon Nookhong. Nutthapat Kaewrattanapit worked together in analyzing the data and comparing the results.

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